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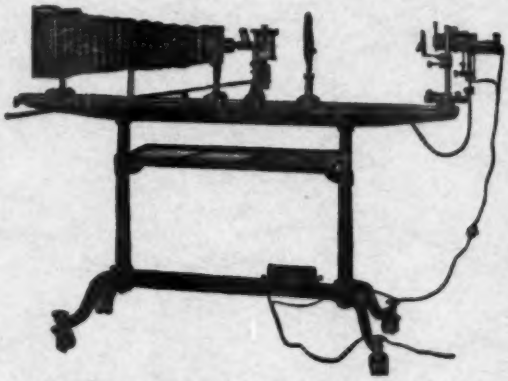

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THE BEGINNINGS OF OPTICAL SCIENCE[1]

BY
JAMES P. C. SOUTHALL

Optics shares with astronomy the distinction of being one of the oldest of the physical sciences. But while astronomy has amassed her vast fortune by long-continued industry and labour, optics has obtained hers in comparatively modern times by sagacious and happy speculations. It is possible that magnifying glasses were used by the Chaldeans about six thousand years ago. The cuneiform characters on the tablets found by LAYARD in the ruins of Nineveh which are now in the British Museum are singularly sharp and well-defined, but so minute in some instances as to be illegible to the naked eye. Specimens of the very implements used to trace these inscriptions were found in the ruins and curiously enough glass lenses were found also, but whether any of the latter have been preserved in the British Museum or elsewhere, I am unable to state.

Exactly what progress in optics was made by the Greeks it is difficult to say chiefly because the original works of the Greek philosophers have not come down to us. The teachings of PYTHAGORAS and even of PLATO have been transmitted more or less inadequately through their disciples, and when we know how even in modern times NEWTON'S enthusiastic followers misrepresented or at least misunderstood his attitude, for example,

[1] Presidential Address before the Optical Society of America, in Rochester, N. Y., October 24, 1921; on the occasion of the Helmholtz Memorial Celebration.

towards the wave-theory of light, it is easy to conjecture that the doctrines of the old Greek philosophers may have fared much worse as they were communicated to a large extent orally from one generation to another. However, we do know in general that the science of the Greeks was based on the introspective and conjectural rather than the inductive and experimental methods, and while this mode of reasoning may be very fruitful in the realm of philosophy, nobody knows better than those who are here present today that it does not help much in unlocking the riddles with which we are confronted in Nature.

The ancient philosophers had satisfied themselves that vision is performed in straight lines and they had fixed their attention upon those straight lines, or *rays*, as the proper object of optics. They had ascertained that rays reflected from a bright surface obey a perfectly definite and extremely simple law, and they were quite familiar with the geometrical properties of mirrors. The art of *perspective* which reached a very high degree of development among the Greeks is merely a corollary from the doctrine of rectilinear visual rays. This art was re-invented in modern times in the flourishing period of painting in Europe, that is, about the end of the fifteenth century.

But the notions of the Greeks concerning vision are vague and confused; although PYTHAGORAS and his followers who believed that vision was due to a material emanation of some kind which proceeded from the object and entered the eye, and that the colour of an object was to be explained party by some peculiarity in the object itself and partly by some subjective process in the eye, were certainly more nearly in accord with modern ideas than EMPEDOCLES who flourished in the following century or ARISTOTLE a generation or two still later.

A few of the ancient writers tried to explain the appearance of a body immersed in a fluid like water, but not much progress was made towards understanding the phenomena of refraction. ARCHIMEDES is said to have published a book "On a ring seen under water," which it is a pity to have lost, because he at least would certainly not have been vague in his geometrical ideas. SENECA remarks that an oar in clear water appears broken and

that apples seen through a glass appear magnified, but he does not offer to throw any light on these familiar observations. The great Alexandrian astronomer PTOLEMY who lived during the reigns of the ANTONINES published a treatise on optics in which he wrote about atmospheric refraction, and he measured carefully the deflection of a ray of light in passing from air into water, for different angles of incidence, without, however, being able to ascertain the law which baffled all subsequent investigators for the next fifteen hundred years.[2]

With him the curtain falls on ancient science, and the intermission covers a long barren period in which it would seem that science had fallen into complete oblivion, indeed, had well-nigh perished from off the earth. The only effective link between the old and the new science is afforded by the Arabs. The dark ages come as an utter gap in the scientific history of Europe; for more than a thousand years there was not a scientific man of note except in Arabia and the lands under Mohammedan rule.

The first real progress in mathematical optics was made by ALHAZEN who died in Cairo in 1038. We must pause to devote some space to him, because, although he was distinguished in many branches of science, he is best known through his optical works which were translated into Latin. ALHAZEN was the first to correct the prevalent misconception as to the nature of vision and showed that the rays of light proceed from the external object to the eye and do not issue from the eye to impinge on the object,

[2] See a recent article on "The Law of Refraction" by Dr. R. A. Houstoun in *Science Progress*, xvi (1922), pp. 397–407; where is given an interesting account of the experimental methods of Claudius Ptolemy for measuring the angles of refraction corresponding to given angles of incidence, with a table showing the actual values which he obtained for air-water, air-glass and water-glass. Further on in the same paper, Dr. Houstoun tells about Kepler's work in this field and gives Kepler's refraction-formula in the form:

$$i = \frac{\mu r}{\mu - (\mu - 1) \sec r},$$

where i and r denote the angles of incidence and refraction, and μ the relative index of refraction; which (he adds) "like the modern formula,

$$\sin i = \mu \sin r,$$

is a one-constant formula." Houstoun subjoins a table of numerical values showing how closely this formula of Kepler's corresponded with the results of his measurements and with the earlier measurements made by Vitellio.

as EMPEDOCLES taught and ARISTOTLE also. His explanations are based on anatomical investigations as well as on geometrical considerations. He showed that the retina is the place where the rays are delivered by the eye in order for the impressions produced there to be conveyed along the optic nerve to the visual centres in the brain. He anticipated HUYGENS and more modern scientists by explaining that the reason why we see single with both eyes is because the retinal images are impressed at corresponding places in the two organs of vision. ALHAZEN likewise pointed out that the sense of sight like our other senses is sometimes misleading, and that we may have optical illusions in consequence of the reflections and refractions of the rays of light. He was perfectly aware that the density of the atmosphere decreases with increase of height, and he deduced the fact that the path of a ray of light coming from a star not in the zenith (to employ one of the many terms with which the Arabians have enriched our European languages) must be curvilinear after it enters the earth's atmosphere, and consequently an observer on the earth will see the star nearer the zenith than it really is. He argued also that we must therefore see the stars for a short time before they have actually risen and after they have actually set. He showed that it is this same atmospheric refraction which accounts for the oval appearance of the discs of the sun and moon near the horizon. The apparent increase of the angular diameters of these bodies in this situation was attributed to a mental illusion arising from the presence in the field of view of intervening terrestrial objects. He showed that the effect of refraction is to shorten the duration of night and darkness by prolonging the visibility of the sun—in fact a "daylight-saving" process which Nature has provided of her own accord. Taking into account atmospheric reflection, ALHAZEN deduced that beautiful explanation of twilight which is to this day the accepted theory. With extraordinary sagacity he applied these principles to a determination of the height of the earth's atmosphere, which he found to extend nearly 58½ miles above the level of the sea—thus anticipating the epoch-making experiments of TORRICELLI and PASCAL by many centuries. "I join," says DRAPER, "as doubtless all natural philosophers will do, in the pious prayer of ALHAZEN,

that in the day of judgment, the All-Merciful will take pity on the soul of ABU-R-RAIHAN, because he was the first of the race of men to construct a table of specific gravities; and I will add ALHAZEN'S name thereto, for he was the first to trace the curvilinear path of a ray of light through the air."

Two centuries glide by before there is another book on optics worthy of note. VITELLIO (*c.*1270 A.D.), a native of Poland, wrote a treatise which is based apparently on the earlier work of ALHAZEN. He compiled a table of the angles of incidence and refraction of light at the surfaces of water and glass, of much greater accuracy than those given twelve hundred years before by PTOLEMY.

But a far more extraordinary man was living and flourishing in England about this same time, and that was ROGER BACON (*c.*1214-*c.*1294) or "Frier" Bacon as he was commonly called in the old English books. Of this eccentric genius whose fame continues to grow we still know too little, except that he must have been born out of due season and was probably centuries in advance of the knowledge of his day. He is credited with having ferreted out secrets of nature which remained hidden from everybody else until our own times. He may have invented spectacles (which appeared in Europe in the thirteenth century) and he is said to have made combinations of lenses which acted like a telescope. Undoubtedly he was familiar with the ordinary properties of a magnifying glass, and he speaks of an instrument by which "the most remote objects may appear just at hand" and with which "from an incredible distance we may read the smallest letters," "and thus a boy may appear to be a giant and a man as big as a mountain," and "so also the sun, moon and stars may be made to descend hither in appearance," "and many things of like sort which would astonish unskilful persons." No wonder he was suspected of being a sorcerer and a practicer of the Black Art. According to an old proverb, "A little knowledge is a dangerous thing," but in the time of ROGER BACON perhaps it was even more dangerous to know too much! At any rate he seems to have been constrained to hide his light under a bushel, and his meaning is often extremely obscure. Like that other famous BACON, the great Lord VERULAM in Queen ELIZABETH'S time, ROGER BACON

had a peculiar fondness, it would seem, for veiling his thoughts in cipher. But the supposed cryptograms of FRANCIS BACON are mere child's play for difficulty as compared with the elaborate system of cipher piled on top of cipher which old "Frier" BACON is said to have invented to protect himself against the persecutions of his fellow-monks who already accused him of being in league with Beelzebub.

BACON was undoubtedly a prodigious scientific genius, but he was an isolated man without any followers. Now again we must wait patiently nearly two hundred years longer before another star of the first magnitude shines through the darkness of the Middle Ages and sheds its lustre on the fair vales of science. This was the renowned LEONARDO DA VINCI (1452–1519), one of the most versatile, gifted and original men who ever lived and certainly the most ingenious and scientific of all artists before or since. The art of perspective which had been cultivated by the Greeks was re-invented by him and placed on a firm basis. It is curious to read now LEONARDO's speculations about the nature of light and to recognize how modern it all sounds.

But the middle of the next century must be taken as marking the real dawn of modern science; for the year 1543 witnessed the publication of COPERNICUS's great work and paved the way for TYCHO BRAHE (1546–1601) and KEPLER (1571–1630). TYCHO re-discovered and explained the phenomena of atmospheric refraction. In an interval in his astronomical studies KEPLER turned to optics with the zeal and industry which characterized everything he did. He gave a very accurate description of the action of the human eye (which will be alluded to again), and "made many hypotheses, some of them shrewd and close to the mark, concerning the law of refraction." His epoch may be said to be the real beginning of optical science in Europe.

GALILEO's long life (1564–1642) overlapped that of KEPLER's at both ends. The two men were kindred spirits though very different in many ways. I cannot pause here to speak in detail of GALILEO's great achievements which are known to all students of science. When the telescope was invented at the beginning of the seventeenth century, the fame of it and of GALILEO's discovery

of the satellites of Jupiter ran round all Europe. I think we can hardly conceive today how men's souls were stirred and their imagination kindled by the almost illimitable prospects which this new instrument seemed to open to their gaze. As we read the writings of a man like HUYGENS (who belonged to the next generation), over and over again we seem to catch some spark of the joyous enthusiasm that possessed those eager men who first employed the telescope and the microscope. On the other hand, GALILEO's discoveries excited a great controversy, as we all know. Some of the most powerful and learned men denounced them as subversive of religion and all the venerable teachings that had been piously handed down through long generations from the time of ARISTOTLE; and not a few of the more orthodox even refused to look through a telescope at all and declared that the celestial appearances which GALILEO was alleged to have witnessed with the aid of this instrument were mere chimeras and hallucinations unworthy of credence and having no corresponding objective reality whatsoever. But KEPLER, the greatest living astronomer in Germany, could not conceal the joy with which he hailed the advent of this wonderful new optical device; he was quick to realize the importance and significance of GALILEO's services and he spared no effort to spread the new knowledge abroad through all the scientific channels of that day. Between him and GALILEO there sprang up an intimate correspondence which forms one of the most interesting records of this great epoch and which was characterized by mutual sympathy and admiration as long as KEPLER lived.

There was also another man in Germany who was keenly interested in the new science and who was likewise destined to have a distinguished share in the advancement of Astronomy and Optics, although outside his own country his fame has not reached the proportions which I think you will agree with me it deserves. He is entitled to take rank alongside of KEPLER and GALILEO as one of the founders of modern optical science. It is because so little has been written about him, that I wish to invite your particular attention to his work. If THOMAS YOUNG who came nearly two centuries later is rightly to be regarded as the father of physiologi-

cal optics, CHRISTOPH SCHEINER (1573–1650) may possibly be called its grandfather in the sense of being the original modern progenitor of this science. When the news of GALILEO's first discoveries was being heralded abroad, young SCHEINER was a student of Mathematics in the Jesuit College at Ingoldstadt. Already in 1603 he had invented the famous instrument which he called a *pantograph* and which is extensively used for making reduced or enlarged copies of drawings and pictures. He read eagerly the accounts which GALILEO published in the Sidereal Messenger (*Nuncius sidereus*); and, while owing to his ecclesiastical environment and predilections he was a little wary of accepting some of the new ideas, he set to work with the greatest zeal and enthusiasm to test for himself this method of investigation. At that time there were only a few telescopes in Germany. KEPLER himself had not been able to verify GALILEO's observation of Jupiter's moons until the evening of August 30th, 1610. But meantime SCHEINER appears to have gotten hold of several of these instruments, because in the first publication of his researches he speaks of having eight tubes of different sizes. With these he straightway began to make a series of observations which were continued with indefatigable labour and which have won for him an honorable place in the history of Astronomy. In 1611 from the tower of the church in Ingoldstadt he turned his telescope towards the Sun whose blinding light was partially obscured by a thin cloud. To his great astonishment he perceived several dark spots on the Sun's disc, which he pointed out to his friend and pupil CYSAT who was present at the time and who likewise recognized them. Thus by one single glance through a telescope the doctrine of the peripatetic school of philosophy about the absolute purity of the Sun had been pierced in the heart! The two observers agreed at first not to speak of this discovery until they had carefully repeated and verified their observations. CYSAT suggested the use of screens of coloured glass which were in common use by mariners in taking the Sun's altitude at sea. Incidentally, it may be remarked that if GALILEO had employed this device in his solar observations, it is possible he might have been spared the blindness which afflicted his old age.

Although SCHEINER's observations of the sun-spots were repeatedly verified by him, he refrained from publishing his results for fear of arousing opposition, but communicated them to a friend in several letters which he permitted to be published in Augsburg in 1612 under the pseudonym of "Appeles latens post tabulam." It was these letters which formed the basis of the bitter priority-controvery afterwards between GALILEO and SCHEINER. I can merely allude here to the furious strife which they waged with each other through all the rest of their lives. A cynic might almost be tempted to quote VERGIL's famous line and ask, "In heavenly minds can such wrath dwell?" GALILEO belittled and ridiculed everything that SCHEINER said or did; and SCHEINER entertained the current opinion among his jesuitical friends and colleagues that GALILEO was a dangerous heretic. As time passed SCHEINER rose to high eminence in the councils of the church; and one wonders what part he took in summoning GALILEO in his old age to appear before the High Court of the Inquisition and make public recantation of his errors. We know that even after GALILEO's trial and humiliation, the stern priest was implacable towards him. Notwithstanding their common interests in the pursuit of truth, they were men of such widely different temperaments and training that it was impossible for either to do justice to the other. SCHEINER was a prodigious and untiring worker, in many ways a typical German, honest and persistent, but apparently without much charm of manner, intolerably prolix in his writings, the kind of fellow I fancy who means well but somehow rubs one the wrong way. GALILEO was just opposite—genial, brilliant, witty, clever, he was never able to recognize the solid good qualities of SCHEINER much less to appreciate the value and importance of his work. By long-continued observations of the sun-spots, SCHEINER established the fact that the Sun turns around an axis, and he found the sidereal period of rotation to be about 25⅓ days, whereas modern measurements make it a little less than 25¼ days.[3]

[3] Neither Galileo nor Scheiner can be said to be the original discoverer of the spots on the Sun, because as a matter of fact these appearances had been noted by the Chinese with the naked eye as long ago as the beginning of the fourth century (301 A.D.). Galileo seems to have seen them for the first time in August 1610, whereas

One of SCHEINER's lectures in the University of Ingoldstadt was on the telescope, which besides giving a minute description of the construction of that instrument contains suggestions about its use for military operations, land-surveying, and particularly in Astronomy. He invented an instrument called a *helioscope* designed especially for solar observations. He also employed a telescope to project the image of the Sun on a screen, as GALILEO did afterwards. At first SCHEINER used in this experiment an ordinary Dutch telescope, but subsequently he found that a telescope with a convex ocular answered the purpose much better. This was apparently the first practical trial of the *astronomical telescope* invented by KEPLER. He also describes a telescope with *equatorial mounting* for keeping the instrument focused at the same place in the heavens, essentially the same as is to be found in our modern observatories today.

At Ingoldstadt and afterwards especially at Innsbruck we find SCHEINER devoting much time to purely optical researches and the nature of vision. The results of these labours were published in 1619 in a very celebrated book called *Oculus sive fundamentum opticum*, of which there were three editions, one published in London in 1652 after SCHEINER's death. This is in every respect a very original and remarkable volume. It is divided into three parts, the first of which contains a description of the anatomical structure of the eye and experiments of various kinds; the second part is a study of the optical performance of the eye and its behaviour towards the rays of light which enter it; while the third part treats of vision, apparent size and questions that are distinctly of the nature of physiological optics. His methods in this treatise are beautifully scientific, every argument being based strictly on experiments.

Scheiner did not see them until March 1611. Meanwhile, however, they had been also observed entirely independently by a certain Johann Fabricius in December 1610, who published a book in June 1611 entitled "Narratio de Maculis in Sole Observatus," wherein the phenomena were minutely described. This was the first announcement in print; but neither Galileo nor Scheiner appears to have had the slightest knowledge of Fabricius's work. Scheiner's first letter on the subject, dated November 12, 1611, was published January 5, 1612. This was followed by Galileo's letter dated May 4, 1612 but not published until the following year.

As early as 1583 a Neapolitan philosopher JOHN BAPTISTA PORTA (1545–1615) who had invented the *camera obscura* compared the eye to it, but he was not sufficiently clear about the anatomical and optical structure of the eye, and he seems to have supposed that the image was received on the crystalline lens. KEPLER was the first of the European scientists to suspect that the retina was the screen where the image was focused, but he could never be sure that this opinion was correct. But SCHEINER established this fundamental fact by actual experiments first on the eyes of sheep and oxen and afterwards in Rome in 1625 on a human eye. Thus he ascertained beyond peradventure that the retina is the peculiar sensory organ of vision, and since he knew also that the retina is a ramification of the optic nerve, he had a very clear idea of how the perception of an external object is communicated through the eye to the brain. The fact that the image on the retina is inverted and that nevertheless the eye perceives the object as erect was correctly explained, as KEPLER had already explained it, by the circumstance that the eye has the faculty of referring each point of the retinal image to the corresponding external point of the object.

However, in order to obtain correct notions about vision, it was necessary, as we can readily understand, to be able to trace the paths of the rays exactly; and to this study SCHEINER devotes all of the second part of his treatise on the eye. He is careful to distinguish the different refrangibilities of the ocular media, comparing the aqueous humour with water in this respect and the lens with glass, whereas he regards the refraction of the vitreous humour as intermediate between the two. He was not able to determine these relations more exactly because even then the law of refraction was still an enigma. SCHEINER himself sought in vain to solve this puzzle, and he made a whole series of painstaking experiments on the refraction of water which were afterwards published in a table giving the values of the angles of refraction for different angles of incidence for every degree between 0° and 90°.

SCHEINER devised a great number of simple but exceedingly ingenious and instructive experiments for showing how the visual

rays arrive at the retina and form an image there. Several of these are classic, one of them in particular which is referred to in all the text-books as "SCHEINER's Experiment." He was one of the first to call attention to the contraction or dilatation of the pupil which always accompanies the act of accommodation.

If I have seemed to dwell too long on SCHEINER's work, it is because I have in mind the occasion which we are gathered to commemorate and because the contributions of this pioneer in Physiological Optics seem to bear very directly on some of the most important work of his great successor in Germany in the nineteenth century.

Some years after the publication of the first edition of SCHEINER's treatise on the eye, but prior to 1626, a Dutch Professor in Leyden named SNELL announced the true *Law of Refraction;* but this law was first published by DESCARTES in 1637 who for a long time enjoyed the credit of having discovered it.[4] SCHEINER and DESCARTES both died in 1650. In addition to his splendid achievements in Philosophy and Mathematics, DESCARTES (1596–1650) was famous also as a natural philosopher and in this department he made many notable contributions to the science of Optics and Vision. He had a remarkable grasp of the *modus operandi* and mechanism of the eye, although some of his optical writings are very vague and unsatisfactory. His great successor HUYGENS alluding to DESCARTES in connection with the theory of the telescope complains that although it is hard to believe it of a man who was so intelligent and well-informed in these matters, nevertheless he got off the track in his demonstrations of the nature and

[4] Eleven years after the death of Snell, Descartes published his *Dioptrics*, in which he announced the law of refraction in terms of sines as we have it now, without mentioning Snell's name. It is said that Descartes had access to Snell's manuscript; Huygens states that he himself had seen the whole manuscript volume of Snell, and that he believed that Descartes had also seen it. "Biot attempts to give the whole glory of the discovery to his countryman Descartes, but the preponderance of opinion is in favour of the view that Descartes' discovery was not independent." While it is probable that this is a question which can never be settled with certainty; it may be added that "Descartes was not generous in speaking of the work of others. In dealing with the invention of the telescope, for example, he does not mention the name of Galileo, and in treating of the rainbow he makes no reference to Antonio de Dominis." See R. A. Houstoun, "The Law of Refraction," *Sci. Prog.*, xvi (1922), pp. 397–407.

effect of the telescope and was sometimes guilty of writing things on this subject to which, to put it baldly, no meaning can be attached!

And now at length the stage is set and the greatest actors of all are ready to step out upon the scene. We have reached the period of that great outburst of optical science in the seventeenth century which followed the invention of the telescope and the discovery of the law of refraction, and which constitutes a kind of Elizabethan Era in the history of scientific achievement. The most illustrious names, of course, are those of HUYGENS (1629–1695) and NEWTON (1642–1727), but clustered around these stars of the first magnitude and forming with them a constellation such as has hardly been seen before or since in the firmament of science are the names of GRIMALDI (1618–1663), who investigated for the first time the difficult and obscure phenomena of Diffraction; ISAAC BARROW (1630–1677), one of the greatest geometers of his day, who had the honour of being NEWTON'S guide and preceptor; ROBERT HOOKE (1635–1703), whose misshapen body was the abode of one of the acutest intellects and who penetrated the secrets of nature with an insight that seems now almost like *clairvoyance;* and ROGER COTES (1682–1716), first Plumian professor of Astronomy in Cambridge University, of whom NEWTON is reported to have said, "Had COTES lived, we might have known something!" If one pauses to think about it, there is something very remarkable about the mutual interaction of pure and applied science in the wonderful story of human progress. For if it is true, especially in this complex modern world of ours with its amazing machinery and useful appliances of all kinds, that the cultivation of pure science stimulates the art of invention and leads to almost bewilderingly new processes of engineering, it is also abundantly manifest that, along with industrial development and the adaptation of the forces of nature, there comes always a higher appreciation of the utility and efficacy of pure science. If one has to look for a needle in a haystack, and has spent many weary hours in the fruitless quest, we may be sure that he will be duly impressed by the discovery that the little bit of steel will leap instantly from its hiding-place at the bidding of a magnet!

A notable illustration is afforded in the history of optics itself when the invention of the telescope and the hope of bringing it to greater perfection were perhaps the chief factors that contributed to the brilliant optical researches and speculations of HUYGENS and NEWTON and their contemporaries. "The practical application," as the late Professor SYLVANUS THOMPSON expressed it, "which we know was in the minds of each of these men, must surely have been the impelling motive that caused them to concentrate on abstract optics their great and exceptional powers of thought."

We all know of HUYGENS as the inventor of the wave-theory of light; but not many of us, I fancy, are familiar with his admirable and lucid explanation of that hypothesis, although it is now quite a long time ago since Dr. CREW (who is one of the speakers on the programme of the HELMHOLTZ Centennial Celebration) published a little volume which contains a translation of some of the most important chapters of HUYGENS'S famous treatise on light (1690) and a few years ago a complete English translation was brought out. HUYGENS had to take account of seven fundamental facts: (1) The rectilinear propagation of light; (2) the interpenetration of two beams of light where they cross each other; (3) the law of reflection; (4) the law of refraction; (5) atmospheric refraction; (6) the finite speed of light, discovered by ROEMER in 1676; and (7) the double refraction of Iceland Spar, discovered by BARTHOLINUS in 1669. As Professor THOMPSON remarks, the insight with which, by the aid of his conception of elementary wavelets building up an enveloping wave-front, HUYGENS succeeded in giving a consistent theory must excite our wonder and admiration. Of course, his wave-theory had to be developed further by YOUNG and FRESNEL long afterwards; but every reader of this work will marvel at the exquisite skill with which he unravelled the intricacies of double refraction in crystals and the anomalies of atmospheric refraction. I have not attempted here to give even an outline of the rich contents of this precious little volume. "HUYGENS was in many respects a notable man. A Dutch aristocrat, a friend in his youth of DESCARTES, he, like the philosopher SPINOZA, was a

grinder of lenses. With his own hands he built in 1665 the telescope with which he discovered the ring of Saturn and the fourth of his satellites."

It was recently my duty and privilege to look through the thirteenth volume of the monumental edition of HUYGENS's Complete Works which was undertaken in 1888 by the Dutch Society of Sciences. This volume which contains HUYGENS's contributions to Dioptrics was issued from the press in 1916 during the great war. With the introduction and notes it comprises more than a thousand pages and is wholly distinct from the treatise on light to which I have already alluded. It is worth noting with what elegance and skill—far in advance of his contemporaries in this respect—HUYGENS derives from the law of refraction the fundamental characteristics of optical imagery. He attached a very definite meaning to the magnifying power of an optical system and he was perfectly familiar with the convenience and importance of the so-called entrance-pupil and exit-pupil which ABBE used to such advantage in his optical studies. A theorem of HUYGENS's which deserves to rank as one of the most beautiful generalizations of theoretical physics and which can be extended to the theory of radiation in general states that, "If an object is viewed through a system of any number of lenses, and if the places of the eye and object are mutually interchanged without disturbing the lens-system itself, the apparent size of the object will be the same as before, and the image will be erect or inverted as before." Every student of Geometrical Optics will recognize at once that this is equivalent to the theorem given by ROBERT SMITH in his *Compleat Systems of Opticks* published in Cambridge in 1738, as the first corollary to be deduced from ROGER COTES's celebrated proposition about the "apparent distance" of an object viewed through a combination of thin lenses. SMITH does not mention HUYGENS's name in connection with this corollary, perhaps inadvertently, possibly also because he considered it as a part of COTES's theorem; for undoubtedly at that time (1738) SMITH must have known of HUYGENS's proposition and in fact he quotes constantly from HUYGENS's writings. There can be no question that HUYGENS is entitled to the priority here, although I never

heard his name mentioned before in connection with this particular matter. HUYGENS himself constantly makes use of this general principle in the solution of special problems, and modern workers will find it serviceable in the same way, just as we often take a short cut in physics by an application of the Principle of the Conservation of Energy.

I have become convinced also by a study of this huge volume that HUYGENS was certainly the first person to recognize clearly the essential fact about a telescope, that the magnifying power of a telescope is equal to the ratio of the focal lengths of object-glass and ocular. He gives three or four proofs of this theorem and attaches the utmost importance to it. It is certain that neither KEPLER nor DESCARTES had grasped clearly this fundamental and distinguishing property of the telescope.

Doubtless few persons are aware nowadays that HUYGENS made valuable contributions to science also in the realm of Physiological Optics. Most physicists are content, so to speak, to deliver radiant energy to the eye and there leave it to its fate; unfortunately, comparatively few of them, like YOUNG and HELMHOLTZ, have thought it worth while to pursue the investigation further and to study the intricate phenomena of vision. Not so HUYGENS; he at least was keenly alive to the fact that at the other end of his microscope or telescope a human eye was adjusted, and that visual perception is the chief thing after all. He was perfectly aware that the magnifying power of the instrument depended on the idiosyncrasies of the eye of the individual. Apparently far more accurately and minutely then DESCARTES or any of his own contemporaries, HUYGENS was acquainted both with the anatomical and optical structure of the eye; and he had the clearest notions about the office of the pupil in the mechanism of accommodation. He perceived that the marvellous power which the eye possesses of seeing either far or near was somehow essentially an affair of the crystalline lens alone; although he wavered in his opinion as to whether the change of focus was produced by a forward displacement of the lens as a whole or by a change in the convexity of the anterior surface or by a combination of both variations. He describes a simple experiment to

show how the pupil of the eye contracts on being exposed to bright illumination. It consists merely in looking through a telescope at a bright background after the eye has been kept in darkness for a suitable time. At the first glance in the instrument the extent of the visible field will be wider than usual, but as the eye gradually becomes adapted to the greater brilliancy, the field rapidly diminishes in consequence of the contraction of the pupil. The essential theory of binocular vision and depth perception was grasped by HUYGENS. He explains in the clearest manner how in order to see an object single with both eyes the two images on the retina must be formed at "corresponding points," just as ALHAZEN had explained it many hundreds of years before. Apparently, HUYGENS failed to perceive that a solid object looks differently to each eye, otherwise he might have anticipated WHEATSTONE and BREWSTER in the invention of the stereoscope. In the first part of the *Dioptrica* HUYGENS gives a description of a "*simplified eye*" formed by two concentric hemispheres of unequal radii. The curved surface of one of these hemispheres corresponds to the cornea and that of the other to the retina of the eye. There is a singular and most striking resemblance between HUYGENS's "simplified eye" and the "reduced eye" conceived by J. B. LISTING in 1845.

After all what impresses the reader most amid all this wealth of material is not so much the theories which are propunded with such rare insight and skill, but the marvellous versatility and resourcefulness of the author and the variety of observations and experiments which underlie the whole and form the solid structure of the edifice itself. To his extraordinary mechanical ability and ingenuity HUYGENS owed much of his remarkable accomplishments; with him to conceive was to execute, no matter what practical obstacles might lie in the way. To this day we read with astonishment of those prodigious "aerial telescopes" with their poles and pulleys which he constructed and mounted with his own hands and with which he made some of his great discoveries in Astronomy. HUYGENS's name is usually associated in our minds with refracting telescopes; but he devoted much study also to reflectors and preferred NEWTON's type of instrument to those

of GREGORY and CASSEGRAIN. Many pages in his *Dioptrica* are devoted to the theory of the compound microscope and his own original microscopical investigations. A whole section of the work is concerned with the intricate theory of spherical aberrations, and he deduced a number of modern formulas on this subject. Chromatic aberrations also received his attention, and he hailed with joy NEWTON'S theory of dispersion which enabled him to subject these aberrations to the same kind of mathematical analysis which he used in the study of monochromatic aberrations.

But enough, I fear more than enough, has been said to give you some idea of the character and scope of HUYGENS'S optical researches; although they represent only a part of his manifold scientific activities. No wonder that he published comparatively little during his busy lifetime! Before he could get his thoughts safely on paper, a whole vista of new ideas begins to distract and fascinate him. New discoveries give ever a new turn to his earlier imaginings, and so he hastens onwards still eager in the pursuit of knowledge when death overtakes him at last at the summit of his great career. *Felix qui potuit rerum cognoscere causas!*

And here I must stop abruptly in the very midst of this thrilling story when, if I have succeeded at all, your interest must be keyed to the highest pitch; because I find I have already exceeded the time which I ought to have on the programme of this meeting. To attempt to give even a brief outline of the history of optics and to abandon the project at the point where Sir ISAAC NEWTON appears upon the scene is like giving the play of Hamlet without the Prince of Denmark. I had hoped to tell not only of NEWTON'S influence and the origin and growth of the new science of Physical Optics as developed by YOUNG (1773–1829) and FRESNEL (1788–1827), perhaps also to dwell a little on FRAUNHOFER'S important achievements particularly in the direction of optical engineering; but above all it was my purpose to remind you of THOMAS YOUNG'S remarkable contributions to Physiological Optics and the work of DONDERS (1818–1889) in Holland which did so much to pave the way for the epoch-making researches and inventions in this

field of science which forms one of the achievements of the great modern scientist who, born in Germany a hundred years ago did so much for the advancement of human knowledge and conferred such blessings on all mankind that with one accord we have assembled here today to do honour to his memory.

COLUMBIA UNIVERSITY

HELMHOLTZ ON THE DOCTRINE OF ENERGY*

BY
HENRY CREW

Mr. Stevenson in one of his best known stories has ably portrayed the character of a man who led two different lives—lives so different, indeed, that one is apt to forget that Dr. Jekyll and Mr. Hyde are really one and the same individual.

Most men, I believe, lead two lives, though not in the sinister sense in which the phrase is employed by the newspapers. A striking illustration of this fact is the late Mr. Burnham, well known to a wide circle of friends as an eminent astronomer; equally well known to another group of friends as the genial and clever clerk of the Federal Court in Chicago. More than once have I met a man who knew about both of these Burnhams without ever suspecting that they were one and the same individual. So widely separated may the two sides of a man be that the public do not recognize them as belonging to the same mind.

A few of the members of this society will recall a volume which appeared many years ago, under the title of "Flatland by A. Square." Here all men were represented as polygons somewhat in the following manner: a thoroughly ignorant and uncultivated man was pictured as a triangle: an ordinary man as a square: a man of better training as a pentagon: a scholar as a hexagon: the perfect man as the circle; women, alas, were limited straight lines. This picture is not entirely bad, for the simple reason that everyone finds most of his own friends many-sided, and responsive to stimuli from many different directions.

The man in whose honor we are gathered to-day differs from most other men, not so much in the number of his sides, as in their high development, in their brilliancy and in their public character. Most men are known to us only through one line of achievement; Helmholtz was pre-eminent as a physicist, but very favor-

*Read October 24, 1921 at the Helmholtz Memorial Meeting, Rochester, New York.

ably known as a physiologist, a mathematician, a philosopher and an artist.

It is my privilege to say something on one of these lines of activity, namely, his earlier work in physics, especially that touching the doctrine of energy.

But before doing this I ought, perhaps to confess, as I have long ago done to your program committee, that my personal acquaintance with Helmholtz was quite brief. It began when I went to obtain his signature, in my *Anmeldungs-Buch*, for the course in Experimental Physics which he offered at Berlin during the winter semester of 1883–4. At the same time I was assigned a private room in the laboratory on the Wilhelm Strasse and the river, not for the purpose of research but simply to make good my lack of laboratory experience. Here the main instruction was given by *Privat Docent*, Dr. Ernst Hagen, later the director of the Second Division of the Reichsanstalt; but Helmholtz had the habit of strolling through the laboratory each day about noon visiting these separate work-rooms, usually inquiring as to what had been done since he was last there, always speaking idiomatic English and nearly always offering some kindly suggestion before leaving. It was here in this laboratory at this time that I first had the pleasure of meeting those two fine spirits, Brace and Keeler.

Helmholtz, as I remember him, had no more striking characteristic than his kindly simplicity. I was confirmed in this view during the summer just passed when in Heidelberg I had the pleasure of meeting Professor Leo Königsberger, the clever scholar and the genial biographer of Helmholtz. I had just asked this eminent mathematician what Helmholtz would think about a certain question. He began his reply with this remark "Helmholtz war ungeheuer einfach!" This simplicity of his showed itself in his dress, in his exposition, in his bearing, and in his method.

Near the end of the semester on going to Helmholtz's office to say good-bye, he was kind enough to give me a commendatory letter to Professor Rowland, which was far more effective than any merit of mine in securing for me a fellowship at Johns Hopkins University; for Helmholtz was one of that very small group of

men, (including Maxwell, Stokes, and Rayleigh), whom Rowland really respected. I saw Helmholtz for the last time at the International Electrical Congress, held at Chicago in 1893. The first session of this body met under the presidency of Elisha Gray, in the Art Institute, then a vacant hall used for public meetings; and it is safe to say that no scientific man, either before or since, has ever received at the hands of Chicago such a spontaneous, warm hearted, and enthusiastic reception as did Helmholtz on this occasion.

You will perhaps allow me to add just one other personal note. My interest in the events of to-day is all the greater because in 1897 at Evanston, we celebrated the 50th anniversary of the appearance of Helmholtz's essay "On the Conservation of Energy." The celebration took the form of a *conversazione* in our physical laboratory where a large variety of transformations of energy were shown experimentally and where President and Mrs. Henry Wade Rogers graciously received the friends of the University.

We come now to the place which this remarkable pamphlet, *Über der Erhaltung der Kraft*, occupies in the history of Physics. But the place which *any* work deserves can be appraised only after one understands the background of ideas which preceded it, and the foreground of results which have followed it. Let us therefore, as briefly as possible, discover the doctrine of energy as it was held just before the time of Helmholtz.

I

Early in the seventeenth century, the idea of force as the time-variation of momentum had been introduced by Galileo and adopted by Newton. Huygens had already introduced the new and appropriate idea of *mass* under the name of "solid quantity." Stevin who died in 1620 had already clearly grasped the principle of virtual work.

It was just here, during the half century, which is possibly the most brilliant known to science, that there arose the great debate between Leibnitz and the Cartesians concerning the nature of *vis viva*, a debate which was finally umpired by D'Alembert and

which resulted in a perfectly clear, workable, and appropriate definition of energy as we know it to-day.

To discover the idea which is appropriate for the description of any particular phenomenon is an extremely difficult matter; and for the very good reason that any physical phenomenon you may choose to select, even the simplest, is extraordinarily complex. Thus the quantitative description of the ordinary process of driving a nail by means of hammer baffled the civilized world for more than twenty centuries. These clouds were first removed in 1668 by the English mathematician, John Wallis, one of the founders of the Royal Society. Wallis pointed out that the decisive factor in the case of the hammer and nail, the battering ram, the base-ball bat, etc. is the product of the force by the time. The idea of an impulse measured by the time-integral of the force, proved to be the appropriate one. This view of Wallis' seemed to support the view of Descartes, that "momentum" is the proper measure of the force in a moving body.

But Leibnitz, just the year before the appearance of Newton's *Principia*, published a little tract with the following title: *A short Demonstration of a Remarkable Error of Descartes' and Others, Concerning the Natural Law by which they think that the Creator always preserves the same quantity of motion; by which, however, the Science of Mechanics is totally perverted.* Leibnitz based his criticism upon two common experiences; (1) that it requires the same "force" (using his terminology and that employed by Helmholtz as late as 1847) to raise a body weighing m pounds through a height of four feet as it does to lift a body weighing 4 m pounds through *one* foot; and secondly, that a body can rise, in virtue of its velocity, only through a height which would produce this same speed by fall. But when a body falls through four feet, it acquires a speed which is just *twice*, (not four times) the speed acquired by falling through *one* foot. If, therefore, the "force" acquired (using the word again in its then current sense) is the same in each case, it follows that it must be measured by the square and not by the first power of the speed.

$$4m \times 1 = 1m \times 4 = work.$$
$$4m \times v^2 = 1m \times (2v)^2 = vis\ viva.$$

The whole discussion finally reduced itself to the question as to what is the proper measure of the "force" of a moving body whose mass is m and whose velocity is v. The Cartesians maintained that mv is the measure of the "force"; while the Leibnitzians were equally confident that it is mv^2.

These differences were first composed when D'Alembert published his great treatise on dynamics in 1743. For, in his introduction to this work, D'Alembert showed that both sides were correct, *vis viva* being the proper measure of a force acting through a certain *distance* and *momentum* the proper measure of a force acting through a certain interval of *time*. Here is the shield with two sides giving us the two fundamental and thoroughly consistent definitions of force which are to-day everywhere used in physics, namely, *the space variation of energy* and *the time variation of momentum*. D'Alembert had taken a great step; the atmosphere was now clear.

The doctrine of energy—though not yet under that name—was now fairly launched. It had long been known that the speed with which a body reaches the foot of an inclined plane, depends, if we neglect friction, only upon the vertical height from which the body starts.

Huygens, as early as 1673, had assumed the constancy of *vis viva* as an axiom. John Bernouilli had coined the expression "conservation of *vis viva*" in 1729; but, as a definite mechanical principle, it appears first to have been used by his distinguished son, Daniel Bernouilli, in 1748.

The next important step in the doctrine of energy was taken by the French Academy in 1775 when by deciding not to entertain any more proposals for perpetual motion they expressed their appreciation of the fact that *vis viva* is uncreatable.

Next comes the word "energy," introduced by Dr. Thomas Young, in 1807, and defined by him as the product of the mass of a body into the square of its velocity. How incomplete this definition is will be seen from the fact that it includes only the mechanical energy of motion and also omits the factor "$\frac{1}{2}$" which had been lost sight of in the scrimmage between the Cartesians and Leibnitzians. This neglect of the "$\frac{1}{2}$" was owing, of course

to the fact that only ratios of energies were discussed by Leibnitz and his followers.

A few years later, in 1826, came the term "work," in its modern sense, suggested by Coriolis to the French engineer Poncelet. From this time on, forms of energy other than those of mechanical motion were rapidly recognized. In 1824, Carnot applied the energy equation for the first time to a non-mechanical phenomenon, viz. heat. Gauss, in 1828, applied it to the energy resident in a liquid surface.

Nevertheless, during all of Helmholtz's boyhood and early manhood, electricity was still the science of two electric fluids: the same was true of magnetism: heat was the science of caloric: light that of the luminiferous ether. The science of Physics was without form and void: it lacked connective tissue; it had no general and controlling principle such as Astronomy and Mechanics had long possessed.

But now evidence was accumulating to show that heat was "a mode of motion," that electrical and magnetic fluids were not unrelated, that light and radiant heat are not only the same sort of thing but that each is a form of energy. These invisible and imponderable substances which had been so lightly called into existence in order to cover our ignorance were being quietly but firmly thrust *out* of existence. In their stead was growing up the new and comprehensive science of energy which from the very beginning was to act as a bond not merely between the various branches of physics, but between all the physical sciences. The new science of energy is therefore something vastly wider than the domain of physics.

But the general equivalence of energy and work, the quantitative transformation of energy, was as yet by no means an accepted fact. It was indeed believed by very few.

Two important events intervened between the time just described and the appearance of Helmholtz's memoir. The first of these is the assumption of the equivalence of heat and work by Robert Mayer in 1842 and his computation of the mechanical equivalent of heat from the difference of the two specific heats of air. The second of these events is the well known experimental

confirmation of Mayer's views by Joule beginning with the year 1843. For Joule not only evaluated the mechanical equivalent of heat but he proved by experiment the truth of Mayer's assumption that the internal work of the expanding air was negligible. These two advances may be described as the establishment of what was later called the First Law of Thermodynamics.

Such is the background of the scene upon which the young army-surgeon of twenty-six appears in 1847, and reads his paper on the *Conservation of Force.* From the title of the memoir, it will be observed that even yet, forty years after Young had proposed the name "energy," the same word is used to denote *force* and the *space-integral of force:* so slow are we all to adopt a new nomenclature even when it is sadly needed to clarify thought. The reason for this delay is, however, not far to seek. The *concept* of energy could not, naturally, assume any great importance until the *principle* of energy had been established. If anyone thinks that accurate definition of scientific terms or the accurate use of them is a small matter, let him recall the terrible riot at Tulsa, Oklahoma, last year which cost many lives and which was precipitated entirely by the inaccurate use of *the word* "assault" by a newspaper reporter.

II

Let us now pass to a brief consideration of the contents of this celebrated essay *Über die Erhaltung der Kraft.*

No sentence in the entire discussion is more characteristic of Helmholtz than the first one in his introduction where he explains that the memoir is intended for physicists, that it is not therefore based upon any philosophical grounds, but upon a single physical assumption, the logical consequences of which he proposes to compare with the experimental laws of physics.

The introduction is mainly occupied with the explanation of this assumption, for which two alternative forms are given, namely, either that perpetual motion is impossible or that all effects in nature are ultimately explicable in terms of attractive and repulsive forces whose intensities depend only upon the distances which separate the particles acting upon each other.

Helmholtz expresses preference for the latter of these equivalent assumptions; but most of his successors, like Carnot and Clapeyron before him, have preferred the former.

Here also is to be found a brief outline of scientific method as Helmholtz understood it at the age of twenty-six. "The final goal," he says, "of scientific theory is, therefore, to discover the final and invariable causes of the processes which occur in nature." At the age of sixty-eight, however, when he annotated a reprint of this essay for *Ostwald's Scientific Series* he adds this note: "It was at a later date that I came to understand that the principle of causality is nothing other than the assumption that the phenomena of nature occur according to law."

This mature judgment of Helmholtz—this clear recognition of the one postulate of physical science—is all the more important because even to-day the idle inquiry as to *why* something happens often distracts the time and energy of the student from the more profitable inquiry as to *how* the thing happens.

The first chapter of the essay is devoted to the enunciation of the well known mechanical principle of the Conservation of *Vis Viva*, namely, that the kinetic energy acquired by a mechanical system in changing from one configuration to another is precisely equal to the work required to bring the system back into the original configuration. The author proceeds to prove that this principle can hold, in all its generality, only when the elementary forces of the material particles are central forces.

In the second chapter, the idea of potential energy is introduced under the name of "sum of the tensions," a quantity which is nothing else than a line-integral of the force. Then the result of the first chapter is generalized into the proposition that, for any system of natural bodies, acting upon one another by attractive or repulsive forces, not depending upon time or velocity, the sum of their kinetic and potential energies must be constant. The nomenclature—*vis viva* and *spannkraft*—is that of Leibnitz; the mechanical ideas are those of Newton; the introduction of potential energy is due to Helmholtz. This theorem is next applied to derive the principle of virtual displacements and thus to establish the whole science of statics. Two noteworthy features distinguish

the expression which Helmholtz here gives the energy principle. The first is that it is quantitative; the second is that it is placed in the form of an invariant similar to that of the conservation of matter.

The third section deals with problems in mechanics to which the energy principle has already been recognized as applicable. These problems are those of the heavenly bodies, falling bodies at the surface of the earth, motions transmitted by rigid solids or by frictionless liquids, the collision of perfectly elastic bodies, waves in elastic media, absorption, and interference.

Helmholtz here suggests that these problems which had hitherto been solved in a great variety of different ways are all capable of solution in terms of energy.

He then employs the fourth chapter in the discussion of a set of phenomena in which it had hitherto been assumed that there was an absolute loss—not merely a waste but an actual destruction—of energy, namely, inelastic collision and friction.

Here the whole question of the mechanical equivalent of heat is raised. Is there such an equivalence? If so, what is the value of the ratio? These questions and others, such as the nature of the heat developed in chemical combination are taken up and solved, as one might expect, in terms of Joule's experimental results and Clapeyron's theorem. His conclusion is that the conservation of energy holds good in every case where the conservation of caloric has, in the past, been found to hold; namely, in the phenomena of conduction, radiation, and change of state. A clear distinction is here drawn between sensible heat as a form of kinetic energy of the molecules and latent heat as work done against the attraction of the molecules. Helmholtz thus presented the modern view of sensible heat as kinetic energy and the so-called latent heat as potential energy which is a form of work, and hence not heat at all.

Attempting to go still farther into the mechanical theory of heat, Helmholtz assumes three kinds of atomic motion as possible; (1) a translation of the center of gravity; (2) a rotation about the center of gravity, and (3) a displacement of the various parts of the atom with respect to one another. But here his

abilities as a prophet fail him and he rests with the possibility of conceiving the phenomena of heat as produced by motion of *some* kind.

One interesting prediction of this chapter is the phenomenon, later discovered by James Thomson, of the lowering of the freezing point of water by increase of pressure. Unfortunately Helmholtz had not, at this time, heard of the work of Robert Mayer, upon which in a note, forty years later, he places a very high estimate.

In chapter five, the discussion is directed toward the mechanical equivalent of electrical processes. First of all the energy of the charged Leyden jar is computed by the standard method, supported by the then recent experiments of Riess; the heat of the discharge is recognized as Joule-heat in the wire joining the two coatings; and this, by means of the mechanical equivalent of heat, is reduced to mechanical units. Here also we find oscillatory discharges definitely predicted and the experimental evidence for expecting them clearly set forth.

The second electrical process considered is that of the galvanic cell. After discussing the two rival theories of the cell, he employs the energy principle to prove that the *e.m.f.* of the cell is proportional to the difference between the amount of heat developed by the combustion of the two metals and by their combination with the acids. The theory of polarization of the galvanic cell next follows; but the phenomenon does not prove amenable to quantitative description in terms of energy. The chapter closes with a demonstration of the fact that the thermo-electric effect of Seebeck is the converse of the Peltier effect.

The sixth and last chapter of the little book starts out by bringing magnetism into the range of conservative forces for the same reason that electrostatics belongs there, namely, each can be explained in terms of attracting and repelling fluids. The author then proceeds with electro-magnetism and succeeds, almost with a stroke of the pen, in deriving Neumann's expression for the *e.m.f.* of induced currents. This is done, as is well known to most of you, by simply equating the energy supplied by the cell during any small interval of time to the Joule-heat plus the change in the kinetic energy of a magnet attracted by the battery circuit. The

invariant sum contemplated by the energy principle is here made up of three items, namely, the chemical energy of the cell, the heat energy in the wire, and the kinetic energy of motion of the magnet. On the whole I should be inclined to reckon this the most brilliant achievement of the entire essay.

The work concludes with a clear and definite suggestion of the applicability of the energy principle to the vital processes of plants and animals, but also with a recognition of the enormous difficulty of actually carrying out such experiments.

From this hasty and imperfect sketch it will be clear to everyone that most of the topics which Helmholtz here takes up are the commonplaces of present day physics—problems whose proximate solutions at least are well known to every advanced student. Why, then, one asks was publication refused in the *Annalen der Physik*. The answer is to be found in the fact that there was as yet no generally accepted explanation of latent heat, no generally accepted definition of electromotive force, and no generally accepted mechanical theory of heat. The theorem that a given quantity of heat is equivalent to a perfectly definite amount of mechanical work is a very different proposition from saying that the heat of a system is simply the kinetic energy of its molecules. Poggendorff's explicit reason, however, for not publishing the paper was that it was not sufficiently experimental in character.

It is always interesting to learn how a genius estimates his own work. I want therefore to read you a paragraph from that remarkable autobiographical after-dinner speech which Helmholtz made on the occasion of his seventieth birthday. A good part of that day had been spent in listening to congratulatory addresses read by representatives of practically all the learned societies of the world. Each of these men emphasized, in his own way, the rare power and intuition of Helmholtz, and the far-reaching importance of the new ideas which we owe to him. In the evening, at the close of the dinner given at the Kaiser Hof in Berlin, Helmholtz rises and, in that modest quiet impressive style of his, tells how simply these ideas came to him, sometimes on first awakening in the morning, sometimes while climbing the mountainside, most frequently in the open air when his body was

rested and his mind fresh. These original ideas never came to the tired brain and never at the library table—*nicht am Schreibtisch.* "The least bit alcoholic drink seemed," he says, "to scatter them." *Die kleinsten Mengen alkoholischen Getränks aber schienen sie zu verscheuchen.*

Among other things he tells just how he came to write the essay now under review. His own words are as follows: "When I was a student in the Medical School in Berlin, I served as assistant in the library, and occupied my leisure moments in acquainting myself with the work of Daniel Bernouilli and D'Alembert. Here I was confronted with the question, as to how the forces of nature must be related in order to render perpetual motion an impossibility; also the further question, *are* the forces of nature, as a matter of fact, related in this way. My purpose in this little book *On Conservation of Energy* was merely to give a critical discussion and an orderly arrangement of the facts for the use of physiologists.

And I should have been perfectly resigned if scholars in this line had said: 'But this is all very well known. What does the young doctor mean by explaining to us in such detail these familiar facts?' To my astonishment, however, the physicists with whom I came into contact gave the matter a totally different reception. They were inclined to deny the truth of the principle and, in their eager fight against the *Naturphilosophie* of Hegel, to regard my work as a bit of fantastic speculation. Jacobi, the mathematician, was the only one who recognized the connection of my ideas with those of Bernouilli and D'Alembert. Jacobi also interested himself in seeing that I was not misunderstood. My young friend E. du Bois-Reymond also gave me his enthusiastic sympathy and support. It was owing to these two men that the recently established Physical Society of Berlin recognized me.

Concerning Joule's work on the same subject, I knew at that time almost nothing, and concerning that of Robert Mayer nothing at all."

III

We pass now to some of the changes which have been introduced into the doctrine of energy since the year 1847.

In 1851, Kelvin supported Helmholtz's view that the electromotive force of a galvanic cell is the mechanical equivalent of the chemical work which is done in the cell by unit current flowing for unit time.

In the same year, Joule set forth his reasons for thinking that the pressure exerted by a gas represents merely the momentum per second delivered to the walls of the containing vessel. In the following year appeared a series of papers by Kelvin in which he gives an exact definition of what is meant by the "heat of a body" —as distinguished from the *total* heat of a body, a definition which marks a long step in advance of Helmholtz, inasmuch as it permits one to assign a definite numerical value to a quantity of heat. The appearance of the first edition of Clausius' *Mechanical Theory of Heat* in 1864 may, perhaps, then denote the complete establishment of the modern view concerning the nature of heat, initiated by Daniel Bernouilli, adopted by Helmholtz and Joule, and made precise by Kelvin.

Leaving to one side the complicated question of the distribution of energy among the various degrees of freedom of the molecule, one may say that something like seventeen years of time was required to secure the general acceptance, by men of science, of the wide principle that heat is merely a special case of the kinetic energy of a system of bodies.

Another important change is in the formulation of the principle in such a way as to make it strictly analogous with the conservation of matter and to render perfectly clear the fact that the conservation of energy is not a mere assumption, not an axiom, not a definition, but an experimental result.

Such a formulation is the following one which follows the style of Kelvin's definition of the heat of a system: *The energy of a material system in any given condition, measured with reference to any other standard condition, is an invariant with respect to time.* Such a statement can be accepted as true only after experiment has shown that the same amount of work is required to bring the system from the given state to the standard state by different routes. Hundreds of such experiments sustain the formulation just given.

To pronounce this principle trippingly on the tongue is a much less difficult matter than to apply it to some particular material system.

Planck has recently pointed out much more forcibly than Helmholtz the difficulties of such an application. For example, one has first to be sure that he knows just what kinds of energy are included in the system; he has then to learn what the quantities are which define the state of the system: he has also to discover whether or not these variables are independent or whether they are related to each other; whether, when one energy element is varied, *all* the other types of energy, or only *some* of them, remain constant. One need only think of the various types of energy represented in a single rain drop; electrical, gravitational, thermal, molecular, atomic, capillary. It is the sum of all these, and perhaps others, which would make up the invariant whose time-variation is zero.

One has also to determine how completely his system is isolated from external influences. And if external influences are at work upon the system, one has then to equate the work done by these external influences to the energy gained by the system. Indeed it is not always an easy matter to define the system one is dealing with. Take, for example, the rain drop of which we were just speaking. Does its surface energy belong to the water or to the water plus the medium surrounding it?

The seat of the energy in any system is a question which Helmholtz did not raise and is one whose answer does not affect the validity of the principle. It is, however, a problem whose answer is engaging some of the alert minds of the present day. The ability to trace energy through its various forms, as we now trace matter through its successive positions, would form a powerful step in advance.

Just such a step as this is precisely what is contemplated in Planck's infinitesimal theory of energy. According to his view, energy can be transferred from the outside to the inside of any closed surface *only through that surface;* in other words there must be some physical process going on *at this surface* during the transfer. These quanta of energy correspond very nearly therefore to

atoms of matter. The conservation of energy becomes therefore a consequence of Planck's theory almost as naturally as the conservation of matter is a result of the atomic theory.

I need, however, hardly call your attention to the fact that the quantum theory of energy—and hence a deductive proof of the energy principle—still leaves much to be desired; and in spite of its fruitfulness is far from being completely established. How, for example, is it to be reconciled with the phenomenon of interference? Where is the seat of the potential energy of gravitational forces?

Let me close these hasty comments by saying that only once since the establishment of the energy principle in the middle of the last century has it ever been seriously called in question. This happened during the brief interval between the discovery of the heat emission of radium by Curie and Laborde in 1903 and the almost simultaneous acceptance of the disintegration theory of matter proposed by Rutherford and Soddy. But it was only for a few brief months that serious-minded men debated the question as to whether radium was a transformer of energy or a magazine of energy. Everything which has since been learned about the atom points to the latter view as the correct one and leaves Helmholtz's essay of 1847 one of the firmest foundation stones in the structure which we call physical science.

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HELMHOLTZ'S CONTRIBUTIONS TO PHYSIOLOGICAL OPTICS*

BY
LEONARD THOMPSON TROLAND

The realm of physiological optics is one which borders upon, and indeed intersects, three great and distinct divisions of scientific effort. Its problems are such as to require for their solution a balanced knowledge of physiology, physics, and psychology, and its results figure almost equally in text-books of these three branches of science. The greatest names in the development of physiological optics have been those of men who in some measure at least combined an acquaintance not only with the facts of physiology but also with those of physics and psychology. Never, however, have the required qualities been so admirably united as in Hermann Ludwig von Helmholtz. A medical man—anatomist and physiologist—by early training, he was also the greatest physicist of his time and is reckoned by historians of the subject as one of the founders of modern psychology. He was at once a powerful mathematician, with a mind keen to theoretical implications, and a skilled disector of organisms with an eye which missed no empirical detail. Men in all of the diverse scientific domains in which he walked recognized him as a master and leader. From such a mind, given the impetus, we might well expect the great contributions to physiological optics and to physiological acoustics which Helmholtz actually made.

The phenomena of color and of tone have fascinated the thought of myriads of investigators, before the time of Helmholtz and since. These phenomena have been studied for many ulterior purposes as aids to artistic endeavor and to visual efficiency in everyday life. Some workers have even exploited the facts under the influence of egotistic ambition. The results which Helmholtz achieved in these fields, however, show that his own inter-

*Read October 24, 1921 at the Helmholtz Memorial Meeting, Rochester, New York.

ests were those of pure scientific understanding; he was obliged in his teaching to expound the principles of science and he found his knowledge of vision and of hearing to be full of lacunae. He hesitated to teach what he himself had not personally verified and his scientific curiosity was aroused by the mysteries which appeared before his penetrating mind. When he actually attacked the problems his method was characterized not only by the thoroughness of the average German investigator but by the directness which distinguishes the English mind, as well as by the succinctness of the French. Helmholtz's great text book on physiological optics covers the entire field with a degree of completeness which has never been approached in any subsequent treatment, and yet in no part of it do we find ourselves wading in words in the search of ideas. Helmholtz's German reads for us not like a foreign tongue but as the clear and universal language of science. (On account of the fact, however, that there are many for whom both the German and the French texts of this great work are necessarily closed books, the wish is aroused that as a fitting tribute to Helmholtz on the centennary of his birth we might before long be able to have prepared and published an English translation.)

Although Helmholtz's interest in his work was purely scientific its fruits were often of immediate practical importance. The *opthalmoscope*, which was the outcome of the earliest of Helmholtz's endeavors in the field of physiological optics, opened the optical shutters of the living eye, so that the ministering hand of opthalmology was no longer necessarily without the guidance of light and vision in attempts to remedy derangements of the visual function itself. This little instrument, although simple in nature, demanded for its conception a recognition of optical principles which had remained obscure to all previous workers. The practical value of the opthalmoscope was at once appreciated by Helmholtz's contemporaries, and he had the rare satisfaction of seeing his invention become quickly the bearer of substantial fruits in opthalmological practice. The medals and praise which came to him as a consequence, however, were received with characteristic modesty.

Helmholtz's earliest work in the field of physiological optics, dealing with the refractive apparatus of the eye, reveals at once that universal characteristic of his endeavors: a desire to penetrate the inner mechanism of nature's processes. Although his measurements of the various constants of the eye were made with the greatest care and accuracy, his fundamental aim was not the mere calibration of the organ for practical purposes but an understanding of its operation in terms of general optical theory, his own experiments being usually so chosen as to have a crucial bearing upon some conception of the ultimate nature of the visual mechanism. By means of two new instruments of his own invention, the phakoscope and the opthalmometer, Helmholtz was able to work out quantitatively the operation of the eye as an optical instrument, or camera, and to make clear the mechanism of accommodation by which it focuses itself upon objects at different distances. He made the first accurate measurements of the radii of curvature of the cornea and of the two surfaces of the lens, showing that in accommodation there is an extensive alteration in the radius of curvature of the anterior surface of the latter. Helmholtz's theory concerning the neuro-muscular mechanism by which this change is accomplished has been subjected to severe criticism by certain subsequent investigators, but as in the case of the majority of other hypotheses which he advanced, no equally satisfactory alternative explanation has appeared. Even if Helmholtz should ultimately be proven in error in any of these cases he will at least retain the credit of having advocated hypotheses to which more than sixty years of labor on the part of others has contributed no substantial improvements or modifications. The general acceptance of Helmholtz's views regarding the dioptric mechanism of the eye is attributable directly to the quantitative nature of his work, to the manner in which he linked up the physical dimensions of the eye with its practical operation through the medium of general optical theory. Had he not been at once a skilled anatomist and a profound student of general optical science this would have been impossible for him.

Helmholtz's thoroughness did not permit him to be content with a demonstration of the grosser optical mechanisms of the

eye. He analyzed closely, with mathematical exactness, many minute details of the entoptic process. His theoretical treatment of acuity data in relation to the diameters of the rods and cones still provides the guiding text of all arguments concerning this topic. His analysis of the part played by diffusion circles and by chromatic aberration in normal vision has only in recent years been surpassed. To the understanding of various refractive defects of the eye he made important contributions, and in his great work on physiological optics he does not neglect to discuss and add to our understanding of the multitude of entoptic phenomena which had been so exhaustively described by his predecessor, Purkinje.

Just as Helmholtz's contributions to our knowledge of the dioptrics of the eye have become permanent portions of science because he was a great student of physical optics, so also have his ideas concerning the reactions of the retina and its attached nervous appendages become a standard content of our text-books because he was one of the pioneers in *nerve physiology*. As the first to demonstrate and actually to measure the velocity of the nerve impulse both in afferent and efferent paths, Helmholtz was well qualified to speculate concerning the neural processes involved in vision or audition, but his speculations here as elsewhere were always guided intimately by experiment. He repeated for himself most of the observations which had been made by his predecessors bearing upon different means by which the retina might be excited, paying particular attention to the various colors which appear as a result of the action of electric currents. In all of his thinking concerning visual sensation Helmholtz was guided by Müller's principle of the specific energy of nerves, according to which the sensation quality is independent of the manner of stimulation and rests wholly upon the identity of the nerve channel which is involved. In his adherence to this principle Helmholtz was not only faithful to the ideas of his own teacher but showed an appreciation of the actual character of the psychophysical nexus which is sadly lacking in many contemporary psychologists and physiologists. Specific energies in

psychophysics and the conservation of energy in physics were the major premises of all of Helmholtz's thinking.

That same interest in the ultimate mechanism of the visual function which led Helmholtz to delve deeply into the dioptrics of the eye caused him to advocate a definite hypothesis as to the physiological operations underlying color vision. It is characteristic of Helmholtz that in this field, where so little was proven with certainty, he did not insist upon a conception uniquely his own, but adopted the fundamental three-color analysis of Thomas Young because it appealed to him as fitting the facts in the case in a maximally satisfactory manner. There has been a great deal of discussion as to the amount of credit which should be given to Helmholtz in connection with the three-color theory; some have insisted that this hypothesis should be named exclusively after Young, while others have called it the Young-Maxwell Theory. The consensus of opinion as expressed in the predominating usage in scientific texts, however, must be regarded as the final judge in such matters, and this insists upon speaking of the Young-Helmholtz Theory. There can be little doubt that the work of Helmholtz with the three-color conception is mainly responsible for its firm establishment in the system of physiological optics. Helmholtz gave the hypothesis definite form in terms of established physiological conceptions, and he worked out its quantitative implications in such a way as to broaden its explanatory function far beyond the status in which it was left by Young.

Concerning the merits of the three-color theory as advocated by Helmholtz, a vast amount of literature has been produced *pro* and *con*. That it should not embrace the whole truth with regard to the color mechanism is inevitable. However, that it schematizes a very considerable body of truth is undeniable. Color vision certainly behaves from many points of view like a three unit system; even Hering's theory, the principal opponent of Helmholtz's hypothesis, conceives only three *integral* mechanisms and the response curves for Helmholtz's three primaries can be converted into those which characterize the primaries of the Hering theory by a mathematical transformation which consists simply in a change of reference axes. Existing theories which

appear to be improvements on the Young-Helmholtz conception actually incorporate all of its essential ideas in one form or another, modifications thus consisting essentially in theoretical accretions which are the normal requirements of augmented data, especially in the biological realm.

Science is indebted to Helmholtz not only for his theoretical development of Young's hypothesis and its application to many new problems but also for a large number of experimental studies upon color. He made clear the difference between the additive and subtractive mixture of colors; with his great mathematical ability he developed the color triangle, as a means of formulating the laws of color mixture, beyond the point to which it had been carried by his predecessors. He devised an elaborate and extremely ingenious instrument for the experimental study of color mixture, which was employed by his pupils König and Dietereci in their classical determination of the three-color sensation curves. Helmholtz is sometimes accused of having neglected the problem of brilliance or apparent brightness in visual sensation, but he devotes nearly one hundred pages to the consideration of this subject in the second edition of his *Handbuch*, discussing the principles of homo- and heterochromatic photometry in great detail. Helmholtz's explanation, based upon suggestions made originally by Fechner, concerning the basis of positive and negative after images has remained as the most plausible one which we know. However, his doctrine that simultaneous contrast is due to purely psychological operations, depending upon "unconscious judgment," has aroused a great deal of opposition and in general is not accepted at the present time, although there is a considerable body of evidence that contrast depends upon a cortical rather than upon an exclusively retinal process. However, the general conception of the influence of a so-called unconscious judgment upon the exact form taken by sensory and perceptual consciousness has apparently found a permanent place in psychological theory.

The universality of Helmholtz's intellect is illustrated in the limited field of physiological optics alone, by the fact that he did not confine himself to the sensory and merely physiological

aspects of the visual process, but extended his investigations over the entire field, including the problems of binocular vision and the theory of space perception. He was the first to show that although the retinal horizontals are parallel to the true horizon, the apparent verticals of the two eyes are inclined to one another at an angle of about two and one half degrees. On the basis of this new data he recalculated the form of the horopter or the locus of points which fall upon corresponding retinal points in binocular vision. He greatly advanced the mathematical analysis of this concept. His investigations in this field led him to considerations of the philosophy of space perception and to this subject also he made significant contributions. His wonderful power as a mathematician enabled him to analyze with great accuracy the movements of the eyes, and he brought to light certain involuntary adjustments of the latter which had not previously been recognized. He showed that the complicated adjustments of the two eyes are all governed by the fundamental principle that they shall see all objects singly so far as possible. Helmholtz's belief that the apparent fusion of the two images in binocular vision is not due to an anatomical confluence of the two optic paths but to a psychic act still finds powerful advocates at the present day. It was in connection with his studies on binocular vision that Helmholtz invented the *telestereoscope*, an instrument for magnifying the stereoscopic effects of objects seen at a distance and one which has had important practical applications. It is interesting that although in the latter period of his life Helmholtz's energies were mainly spent upon purely physical problems he was constantly recurring to visual questions, some of his last papers dealing with sensory matters.

It will not be out of place to say a word in this address concerning Helmholtz's contributions to physiological acoustics. His book on sensations of tone is scarcely less of a masterpiece in its own field than the *Handbuch der physiologischen Optik*. Like the latter it has found no rival in sixty years time. Helmholtz's investigations into the processes of audition were stimulated not only by his general physiological interests, and the analogy with vision, but also by his interest in music, being himself an accom-

plished pianist. The conceptions of timbre, harmony and the like with which he dealt were not to him mere theoretical ideas but were vivid realities of his experience, yet his attack upon the problem was that of the scientist rather than of the artist; his efforts passed with no embarrassment from a laboratory dissection of the delicate structures of the ear, through a mathematical analysis of their mechanics to applications of the results in the realm of music. It is small wonder that in the domain of physiological acoustics the peer of Helmholtz is yet to be found. His hypothesis concerning the operations of the cochlea was the first to provide any intelligible idea of the mechanism of auditory response, and although numerous rivaling theories have subsequently appeared, Helmholtz's views still hold their position as the most plausible.

When one with ambitions in ever so small a field of scientific effort surveys the work of Helmholtz he must necessarily feel stunned by the magnitude of this great man's achievements. He is faced by a mind masterful in the most diverse domains; in physics, in biology, in psychology, in mathematics, and even in philosophy, an accomplished specialist at one time in far separated, minute, subdivisions of scientific endeavor. Very few of us can hope to parallel his accomplishment even in a very restricted field, yet to recognize the greatness of Helmholtz and to assimilate at least to a certain extent his methods and aims should be one of the greatest aids possible toward a continuation of the character of work which he himself accomplished. On this centennial of Helmholtz's birth we should not only eulogize his name but we should turn back to a renewed study of his monumental works. How many students of visual physiology are there at the present time endeavoring to solve problems the solutions of which are to be found in the *Handbuch der physiologischen Optik?* How many are there working without the knowledge which this great book silently offers as a guide in their endeavors. How many are there, moreover, who are dealing with fictitious or impossible questions the absurdity of which would be apparent to any close student of Helmholtz? Let us, who are living investigators of the phenomena of vision, resolve to read and compre-

hend from one cover to the other that great Bible of physiological optics which Helmholtz so painstakingly prepared for us. By so doing we shall make a tribute to our great leader, of the sort which he himself would most keenly appreciate.

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REMINISCENCES OF HERMANN VON HELMHOLTZ*

BY
M. I. PUPIN

After graduating at Columbia in 1883 and after studying Mathematics and Dynamics at the University of Cambridge, England, for three semesters, I went to Berlin to study Physics with Helmholtz. Kirchhoff was still living and I heard him lecture on the Theory of Electricity during the Winter semester 1885–86. He died during the early part of 1886.

Helmholtz was then the director of the Physical Institute of the University of Berlin. His title, conferred upon him by the old Emperor, was Excellenz, and the whole teaching staff of the Institute stood in awe when his name was mentioned. The whole scientific world of Germany, nay the whole intellectual world of Germany, stood in awe when the name of Helmholtz was mentioned. Next to Bismarck and the old Emperor he was at that time the most illustrious man in the German Empire.

I had letters of introduction to him from President Barnard of Columbia University and also from Professor John Tyndall of the Royal Institution. Professor Arthur Koenig, the right hand man of Helmholtz and the senior instructor in the Physical Institute, took me to the office of Excellenz von Helmholtz and introduced me as Herr Pupin, a student from America. Koenig bowed before his master as if he wished to touch the ground with his forehead. I bowed American fashion, that is, with a bow of the head which did not extend below my shoulders, the same kind of bow which was practiced at the University of Cambridge at that time, and called it the Anglo-Saxon bow; it is entirely different from the Teutonic bow. Helmholtz seemed to notice the difference and he smiled a benevolent smile; the contrast evidently amused him. He had much of Anglo Saxon blood in his veins and was quite familiar with the manners of English polite society.

*Synopsis of an address presented October 34, 1921 at the Helmholtz Memorial Meeting, Rochester, N. Y.

He received me very kindly and showed deep interest in my plans of study. His appearance was most striking; he was then sixty-four years of age, but looked older. The deep furrows in his face and the projecting veins on the sides of and across his towering brow gave him the appearance of a deep introspective thinker, whereas his protruding scrutinizing eyes marked him a man anxious to penetrate the secrets of nature's hidden mysteries. The size of his head was enormous, and the muscular neck and huge thorax seemed to form a suitable foundation for such an intellectual done. His hands and feet were small and beautifully shaped and his mouth gave evidence of a sweet and gentle disposition. He spoke in the sweetest of accents and little, but his questions were direct and to the point. When I told him that I never had an opportunity to work in a physical laboratory and paid exclusive attention to theoretical physics, he smiled and suggested that I should make up this deficiency as soon as possible. "A few experiments successfully carried out usually lead to results more important than all the mathematical theories" he assured me. He then requested Professor Koenig to map out for me a suitable course in the laboratory and to look after me. Koenig did it and I shall always be grateful to the kind little man with the bushy red hair and distressingly defective eye sight. Helmholtz was always mellow hearted to Koenig, because, I think, Koenig reminded him of his own son Robert, who was deformed in hand and foot and back, but had the magnificently shaped head of his great father.

During my first year's study in Berlin I attended Helmholtz's lectures on experimental physics. They were most inspiring, not so much on account of the many beautiful experiments exhibited there, as on account of the wonderfully suggestive remarks which Helmholtz would drop every now and then under the inspiration of the moment. At these lectures Professors Koenig and Dieterici usually assisted; they attended to the purely academic elements of the experiments, whereas laboratory mechanician Noehde and Institute janitor Kabisch handled the apparatus and took most of the responsibility for their prompt operation. Helmholtz threw the searchlight of his giant intellect upon the

hidden meaning of the experiments. These lectures were attended not only by students in physics, mathematics, and chemistry, but also by medical students and army officers. The official world, and particularly the army and navy, paid close attention to what Helmholtz had to say, and I have much reason to believe that they consulted him at every step. I have been often called upon to correct the opinion that Helmholtz was a pure scientist par excellence. There is no doubt that his great work dealt principally with fundamental problems in scientific theory and in philosophy, but there is also no doubt that he was intensely interested in the applications of science to the solution of problems which would advance the industries of Germany and of the whole world. His earliest career is associated with his invention of the opthalmoscope. His design and disposition of the coils in a galvanometer are well known. The optical glass of Germany was developed by his former students who led the world in geometrical optics, a part of Physics to which Helmholtz devoted much attention in his younger days.

One day I was on my way to the Institute; in front of me walked a tall German army officer, smoking a big cigar. When we reached the entrance to the Institute the officer stopped and read a sign which said: Smoking is strictly forbidden in the Institute building. The officer threw his cigar away and walked in. I recognized Crown Prince Frederick in the officer. I watched his footsteps and saw that he entered Helmholtz's office and stayed there over an hour. He undoubtedly consulted the great physicist on some scientific problem which was then interesting the German Army and Navy. Many a time I saw the great electrical manufacturer, Werner von Siemens, call at the office of Excellenz von Helmholtz and every time I saw him I grew more anxious to meet him personally. As a proof of great personal favor, Helmholtz gave me a letter of introduction to his distinguished friend, who received me most kindly and gave me an official to guide me through his great electrical plant in the Markgafenstrasse, in the heart of Berlin. Little I thought at that time that some day these great works would have a special department for the manufacture of some of my inventions, and that I would have the very honor-

able distinction of knowing personally the distinguished sons of the great Werner von Siemens.

The great Reichsanstalt in Charlottenburg, the first President of which was Herman von Helmholtz, is a memorial to the warm friendship which existed between Helmholtz and Siemens. It was a gift of Siemens to the physical science of Germany. This gift bore magnificent fruit. The study of black body radiation, first undertaken here by Wien, Lummer, and Pringsheim, under the direction of Helmholtz, led to Wien's formulation of the displacement law and to Planck's formulation of the Quantum theory. In return for this splendid gift Helmholtz gave to the Siemens works his pupils, Rapps, Francke, and Ebeling under whose directorship the Siemens works have grown to wonderful proportions, being one of the largest and best organized electrical plants in the world.

During my three years stay in Berlin Helmholtz was not engaged personally in experimental research. This was done under his direction and upon subjects suggested by him to candidates for doctor's degree and by his former students. Thus Koenig and Brodhun conducted experimental researches connected with Helmholtz's theory of colour and the theory of vision. Menzel, Rapps, and Max Wien were experimenting in accoustical vibrations, and Hertz was busy with the electrical problem which Helmholtz assigned to him, the problem, namely, of electrical vibrations. Hertz finished his experiments conducted on somewhat different lines than originally suggested by Helmholtz, and in the early part of 1887 he sent a preliminary report to Helmholtz, who discussed it at the next meeting of the Physical Society. I was present at the meeting and shall never forget the enthusiasm with which Helmholtz discussed Hertz' remarkable results. He congratulated German science for the distinction which fell to its share when one of its sons first revealed to the puzzled world the real meaning of Maxwell's great Electro-magnetic Theory. Kirchhoff, one of the great scientific stars of Berlin, had died a year prior to that, but DuBois Raymond, the great physiologist, and Hoffmann, the great chemist, were still living and were present at that historical meeting. Among the young men who

have since achieved distinction and who were present at that meeting should be mentioned: Willy Wien, Max Wien, Rubens, Pringsheim, Lummer, and Arthur Gordon Webster.

But although Helmholtz did not engage personally in experimental research he was deeply engaged in theoretical research. Thus in 1886 he published his great theoretical study entitled: "Thermodynamics of Chemical processes." It was a splendid supplement to Willard Gibbs' epoch making studies and with it forms the broad foundation of modern physical chemistry, which at that time became so popular among the physicists on account of the remarkable experimental results obtained by Arrhenins, Nernst, van't Hoff, and Ostwald. Helmholtz's theoretical work was the basis of my Doctor dissertation on "Osmotic Pressure and its relation to Free Energy." His advice to me to avoid strong statements which would involve me in a heated discussion with Nernst and Ostwald was the advice of a kind father to a pugnacious son. Towards the end of his life Helmholtz made several attempts to explain Electrical and Thermal Phenomena, by generalized Dynamics, paying much attention to the Principle of Least Action and to the Hamiltonian Principle. The question of the ether also received much attention, but with the exception of his simple and elegant Electromagnetic theory of dispersion one cannot say that these theoretical discussions had received from Helmholtz their last finishing touches. In 1893 he visited the United States as an official representative of the German Government at the Chicago Electrical Congress, at which he presided. He and Frau von Helmholtz spent one Sunday on the Jersey shore as my guests. I asked him on that occasion whether he intended to continue his theoretical studies in electrodynamics to which he had given so much attention and he said that he did not, having come to the conclusion that Maxwell had spoken the last word in the electrical theory concerning phenomena which were known at that time. Additional experimental results, with electrical motions exhibiting large accelerations were badly needed, he said. Had he lived two years longer he would have seen Roentgen's discovery and the discoveries which followed in the wake of the Roentgen rays. These would have shown him the great accelerations for which he like a prophet and a seer was looking.

During his three months stay in America in 1893 he was fêted and feasted everywhere. During a reception in his honor in the Columbia University library, an incident occurred which should be recorded here. Hundreds of distinguished people of New York attended and waited for their turn to shake hands with the great physicist. A mathematical physicist, well known at that time for his great erudition and remarkable lack of tact and common sense, begged me to introduce him to Helmholtz and to be sure to mention his accomplishments which I finally did and then stood by and watched. Helmholtz was suddenly attacked by the question: "What is your opinion, Excellency, about the negative curvature of space?" Helmholtz answered: "Just now I have no opinion on that subject," and his expression became a perfect blank. I rushed to his rescue by saying: "Excellency, Frau von Helmholtz is waiting for you with a cup of tea," and he followed me, getting rid of the negative curvature philosopher rejoicing in the feeling that he formulated a question which even the greatest living physicist could not answer. A few days later I met this philosopher and he informed me that in his opinion Helmholtz had the expression of a rather dull man. "Yes," said I, "Helmholtz always reflects the mental image of the people who are addressing him." There was more truth than jesting in what I said. I have often watched Helmholtz at receptions given by Frau von Helmholtz in her Berlin house. She would always assign a definite place for his Excellency and there he stood for several hours without budging, listening with stoical resignation to all kinds of remarks addressed to him by the guests. When a clever and interesting person addressed him his face looked as radiant as the Eastern summer sky at sunrise. When addressed by a stupid bore his face would remind you of a lead colored sky devoid of color and contour. A squad of young research men of the Institute were always present at these receptions and when we noticed in Helmholtz's face signs of despair one of us would step in and displace the bore who was subjecting Helmholtz to a mental torture.

The face of Helmholtz reminded one very much of the face of Bismarck. It had that massive and almost overpowering strength

which Bismarck's face had, but there was in it also the luminous light of spirituality which Bismarck's face did not possess. Helmholtz never prepared his lectures on theoretical physics; he worked his subject out during the lecture and thus gave us students a chance to see the expression in that magnificent face when the mind of the master was in full action. Knaus in his famous Helmholtz portrait, now in the National Gallery in Berlin, caught that expression, but it cannot be found in any other picture of Helmholtz. During his return voyage from America in 1893 Helmholtz met with an accident on the steamer, injuring his head. It looked like a slight injury, but in the following year he died from the effects of it. His death ended one of the most glorious scientific careers which the world has ever seen.

COLUMBIA UNIVERSITY

A THEORY OF INTERMITTENT VISION

BY
HERBERT E. IVES

INTRODUCTION

Through the work of Porter,[1] Kennelly and Whiting,[2] Luckiesh,[3] Ives and Kingsbury,[4] and others, there have been determined relationships between critical flicker frequency and the intensity and wave-form (intensity distribution with time) of the illumination of the observing target. These relationships are comparatively simple, the frequency occurring as a logarithmic function of the other variables. They constitute probably the most complete and definite data we have on the time factor in visual response. This being the case, it would appear that from these relationships it should be possible to form an idea of the kind of mechanism and processes involved in vision. It is the purpose of this paper to describe a mechanism and processes, which, on certain not improbable assumptions, would behave toward intermittent illumination substantially as does the eye.

REVIEW OF EXPERIMENTAL DATA

The theory to be developed is based on data already published, the only new data being certain sets of confirmatory observations which are used in the illustrations. The method employed to obtain the new data presents no points of striking novelty and need not be described.

The principal *high intensity* critical frequency relations as obtained by the experimenters quoted may be summarized as follows:

1. For all wave forms, critical frequencies plot against the logarithm of illumination as parallel straight lines[1, 2, 3].

2. If the flicker range is varied, the range factor (amplitude) enters as a multiplier of the illumination.[2]

[1] Porter, Proc. Roy. Soc. *113*, p. 347, *120*, p. 313, *136*, p. 445.
[2] Kennelly and Whiting, National Electric Light Assn., 1907 convention.
[3] Luckiesh, Physical Review *4*, 1, p. 1; 1914.
[4] Ives and Kingsbury, Phil. Mag. April 1916, p. 290.

3. If the ratio of exposure to obscuration is varied, critical speeds are to a close approximation the same for complementary openings[1, 4].

These relationships may be summarized symbolically as below: where ω is critical speed in cycles per second, I is illumination, or more properly brightness;

for 1, $\omega = a + b[\log I + \log F]$ where F is some constant characteristic of the wave form.....(1)

for 2, $\omega = a' + b' \log I\ \mathfrak{a}$ where $\mathfrak{a}$ is the amplitude (a special case of (1)).....(2)

for 3, $\omega = a'' + b'' \log I + f[\Phi(1-\Phi)]$ where Φ is the fraction of the period during which exposure occurs.....(3)

At *very low intensities*, using short wave radiation (scotopic vision) critical speeds are independent of intensity and vary only with the waveform.[5] The experimental results may be summarized in the empirical formula

$$\omega = c \log \frac{2W}{\delta} \qquad \text{.....(4)}$$

where W is the coefficient of the first periodic term of the Fourier expansion representing the waveform, divided by the mean value of the stimulus; c and δ are constants.

PREVIOUS THEORETICAL TREATMENTS

The writer knows of but three previous attempts to correlate these critical speed relations with specific theories of visual response. Their salient points may be briefly mentioned here, but the original papers should be consulted for details.

Kennelly and Whiting[2] state that their observations "conform substantially to the Weber-Fechner law of sensation and stimulus," that is that

$$\Delta S = \frac{\Delta I}{I}$$

[5] Ives, Critical Frequency Relations in Scotopic Vision, J. Opt. Soc. Am. May, 1922, p. 254.

and are led from this to speak of the "sensation of visibly flickering illumination" as following the same law with relation to stimulus as does the sensation due to steady illumination. While this postulation of a "flicker sensation" introduces a conception exactly in accordance with the experimental facts (being but a restatement of them) it is questionable if it gives real aid in picturing a visual mechanism.

L. T. Troland[6] assumes a process of decomposition and recomposition which leads, in the case of equal light and dark intervals, to the experimentally obtained logarithmic relation. Upon applying the same line of reasoning to the case of unequal exposure and obscuration it will be found that the critical speed should be, for any illumination, a linear function of the fraction obscured instead of the symmetrical function of opening required by experiment and symbolized in (3).

The third derivation to be noticed is that by the writer and Mr. Kingsbury.[4] In this the decrease of amplitude of a transmitted impression is ascribed to the process of conduction according to the Fourier diffusion law; change of illumination is supposed to cause a change in the diffusion constant. This theory indicates that the $\omega - \log I$ plots for different openings should be considerably inclined to each other, and that the small opening critical speeds should be higher than the complementary large opening speeds. These predictions are qualitatively verified, but the dissymmetry is less than the theory calls for.

NEW THEORETICAL TREATMENT

In seeking for a theory of visual response which would lead to the critical frequency relationships, the effort has been to make the theory harmonize as closely as possible with the probable nature of the visual process, as indicated by lines of study other than that of critical frequency. Thus it is probable from several lines of evidence that the relation between stimulus and response is approximately *logarithmic;* such a relationship should be included in a visual response theory. As another, and, indeed much

[6] Troland, Am. Jn. Physiology *32*, [May] p. 8; 1913.

more certainly established fact is the behavior summarized by Talbot's law, that the response (sensation) shall be the same for the same mean illumination, no matter whether this is steady or intermittent, provided the speed of intermittence is so high that flicker vanishes. The harmonizing of these two phenomena of response presents considerable difficulty.[7] More specific ideas of visual response lately current[8] indicate the initial reaction to be a photochemical one, followed by some relatively slow process of conduction.

The new theory here developed postulates three steps in the process of the perception of flicker. The *first step* consists of a photochemical (photoelectric) reversible reaction of such a nature that the equilibrium value under steady illumination is proportional to the logarithm of the stimulus. The *second step* consists of a conduction process, according to the Fourier diffusion law, as developed to cover conduction accompanied by leakage, or re-composition of the diffused substance. The *third step* consists in a perception process in which the criterion for perception is that the time rate of change of the transmitted reaction must exceed a certain constant critical value. These three steps will now be treated in detail.

The initial reaction is assumed to be of the kind occurring in photoelectric cells with liquid electrolytes (Becquerel effect). In these cells, as shown for instance by the work of Goldmann[9] on metal electrode cells containing dye solutions, the primary emission of electrons is proportional to the intensity of the illumination. As the illumination continues there is an accumulation of charged ions which are continuously being neutralized, so that an equilibrium is reached under illumination which is, over a considerable range, proportional to the logarithm of the illumination.[10] On the removal

[7] See Drysdale, Proc. Optical Conv., p. 173, 1905.

[8] Hecht, Science, April 15, p. 347, 1921.

[9] Goldman, Ann. der Physik, 27, p. 449, 1908.

[10] The electrical behavior of these photoelectric cells under illumination is strikingly like that of the excised eye, as studied by Waller and other. Notable resemblances are shown in the preliminary negative response on commencement, and terminal positive twitch on cessation of illumination, and in the reversal of the reaction with age. Bose ("Response in the Living and non-Living," p. 169) remarks "there is not a single phenomenon in the responses, normal or abnormal, exhibited by the retina, which has not its counterpart in the sensitive cell constructed of inorganic material."

of the illumination, the charge (potential) declines to its dark equilibrium. The potential variation of such a cell under intermittent illumination by a sector disc of open fraction Φ and period τ, is represented by a saw-tooth variation around the mean ($\propto \log I\,\Phi$) rising during the time $\Phi\tau$ and falling during the time $(1\text{-}\Phi)\tau$. The higher the speed the smaller the amplitude of variation, and the more nearly the two slopes of the saw-tooth potential variation plot approximate to straight lines.

In order to study this kind of reaction quantitatively, we may set down as representing the facts sufficiently closely, the following equation

$$c\frac{d\Theta}{dt}+be^{\Theta}=f(I,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where c is the capacity of the "cell," Θ is the potential (concentration of ions), b a constant, e the logarithmic base, $f(I, t)$ the mathematical expression of the time and intensity distribution of the illumination. The equation states that energy is being received by the system at a rate represented by $f(I, t)$, is being stored (capacity factor), and is being lost or neutralized in such a way that if a steady state obtains $(\frac{d\theta}{dt}=o)$

$$be^{\Theta}=f(I,t), \text{ or } \Theta=\log\frac{f(I,t)}{b} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

The general solution of (5) may be obtained[11] by multiplying through by $e^{-\Theta}$; doing this, and substituting y for $e^{-\Theta}$ we get the equation

$$c\frac{dy}{dt}+y\,f(I,t)=b \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

of which general solution is

$$y=\frac{be}{c}^{-\frac{1}{c}\int f(I,t)dt}\left\{\int e^{+\frac{1}{c}\int f(I,t)dt}dt+\text{const.}\right\} \quad \ldots (8)$$

Before using this solution to get the relations between I, Φ and ω for particular values of $f(I, t)$, it is of importance to determine

[11] I am very greatly indebted to Mr. T. C. Fry for assistance in the mathematical work which follows, and for helpful discussions of the general problem.

whether the kind of reaction represented will take care of Talbot's law. To do this we note that any periodic stimulus such as that resulting from the interposition of a sector disc rotating at uniform velocity before a light source, may be represented by the general equation

$$f(I, t) = I\Phi + A \sin \omega t + B \sin 2\omega t + C \sin 3\omega t + \text{etc.} \quad \ldots\ldots (9)$$

where $I\Phi$ is the mean value of the stimulus. Introducing this into (8), and making $\omega = \infty$, we get, since $\int A \sin \omega t = -\frac{A}{\omega} \cos \omega t$

$$y = \frac{b}{c} e^{-\frac{1}{c}\int_o^t I\Phi . dt} \left\{ \int e^{+\frac{1}{c}\int I\Phi dt .} dt + K \right\} \quad \ldots\ldots (10)$$

$$= \frac{b}{I\Phi} + K' e^{-\frac{I\Phi t}{c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

so that after such time as the second term becomes negligible

$$e^{\Theta} = \frac{I\Phi}{b} \text{ or } \Theta = \log \frac{I\Phi}{b} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

But from (6), this is the value of Θ for a steady illumination of value $I\Phi$. Hence Talbot's law is obeyed. For speeds less than infinite, but still large, the fluctuations will be to either side of $\Theta = \log \frac{I\Phi}{b}$, and Talbot's law will be more and more widely deviated from, with decreasing speed, as the difference between the mean position of the fluctuating potential corresponding to $I\Phi$ becomes greater and greater.

We may now proceed with the solution of (8). For the present we shall consider only the case of the "square topped" stimulus, of value I between $t = O$ and $t = \Phi\tau$, and of value zero between $t = \Phi\tau$ and $t = \tau$, (i.e. for time $(1 - \Phi)\tau$). For this (5) becomes

$$c\frac{d\Phi}{dt} + be^{\Theta} = I\Phi + \frac{2I}{\pi} \{ \sin \pi\Phi \cos \omega t + \tfrac{1}{2} \sin 2\pi\Phi \cos 2\omega t + \tfrac{1}{3} \sin 3\pi\Phi \cos 3\omega t + \text{etc.} \} \quad \ldots\ldots (13)$$

where ω is the frequency in cycles per second $= \frac{1}{\tau}$. The exact

solution of (13) is obtained by inserting the right hand member in (8) for $f(I, t)$, and performing the integration for a series of values of I, Φ and ω. From plots of the values obtained it is then possible to find the desired factors, such as amplitudes, and slopes of the reaction. However, with the knowledge that the amplitude of the reaction must be kept small, (for Talbot's law to hold), we may obtain an approximate solution without resorting to graphical methods, as follows:

Consider a high speed of alternation with the potential, Θ, varying by a small amount to either side of the value given by $e^{\Theta} = \frac{I\Phi}{b}$. Now consider what happens immediately after a dark sector has covered the light. The potential will fall according to the relation.

$$c\frac{d\Theta}{dt} + be^{\Theta} = 0 \quad \ldots\ldots(14)$$

Now since for small fluctuations be^{Θ} differs but little from $I\Phi$ we may write

$$c\frac{\Delta\Theta}{\Delta t} = -I\Phi \quad \ldots\ldots(15)$$

or

$$\Delta\Theta = -\frac{I\Phi}{c}\Delta t \quad \ldots\ldots(16)$$

$$\text{Noting that } \Delta t = (1-\Phi)\tau = \frac{1-\Phi}{\omega} \quad \ldots\ldots(17)$$

we get for the amplitude of the drop in potential before the light is again thrown on

$$\Delta\Theta = \frac{1}{c}\ \frac{I\Phi(1-\Phi)}{\omega} \quad \ldots\ldots(18)$$

We have then, as the result of the intermittent illumination of our photoelectric cell, a fluctuating potential in which, for large values of frequency, the amplitude is proportional to the intensity, and inversely as the frequency.[12] The shape of the potential-time

[12] It will be noted that this result follows, whatever the form of the re-composition function. The choice of be^{Θ} for this function is in deference to the generally accepted idea of a logarithmic response to a steady stimulus. The farther the re-composition function departs from a simple direct proportionality to the reaction strength the higher must be the frequency to insure Talbot's law holding. It is a fact of experiment that this law already holds at the critical frequency for flicker disappearance.

curve is that of a saw-tooth, varying from a long rise and short drop ($\Phi > 1-\Phi$) to a short rise and long drop ($\Phi < 1-\Phi$). (Fig. 1, *b*.)

It is obvious that this process alone does not yield the typical (logarithmic) critical frequency relationships. We proceed now to the second step, the conduction process. We assume *as our stimulus*, the potential or concentration of decomposition products of the photochemical reaction varying in the manner just described. How will a typical conducting medium transmit this stimulus?

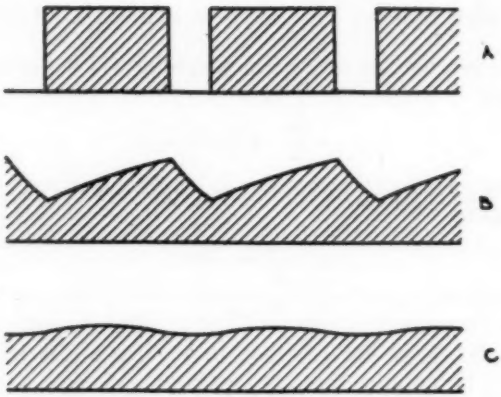

FIG. 1

Successive steps from stimulus to final reaction
(a) Square-topped stimulus
(b) Reaction of photoelectric cell
(c) Reaction at far side of conducting medium.

The general expression for conduction according to the Fourier law is

$$\frac{\partial \Theta}{\partial t} = K\frac{\partial^2 \Theta}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

where Θ is the potential (concentration) at a point distant x from the stimulated surface, and K is the diffusivity. This expression assumes no loss or re-composition of the conducted material. If we assume a leakage or process of neutralization we may modify (19) to[13]

$$\frac{\partial \Theta}{\partial t} = K\frac{\partial^2 \Theta}{\partial x^2} - \mu\Theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

[13] See Preston's "Heat," 2d Ed., p. 654.

placing the loss, as the simplest possible assumption, proportional to the potential.

This equation is to be solved by introducing as the boundary condition at $x=O$, the proper expression for the saw-tooth stimulus, of which (17) is the fluctuating part. This expression consists in general, of a constant (S), representing the minimum value of the stimulus, to which is added the Fourier expansion of the fluctuating portion. For the symmetrical saw-tooth ($\Phi=1-\Phi$) the Fourier expansion of the fluctuating portion of amplitude $\Delta\Theta$, is

$$\frac{\Delta\Theta}{2}+\frac{\Delta\Theta}{2}\cdot\frac{8}{\pi^2}(\sin\omega t-\frac{1}{9}\sin 6\,\omega t+\frac{1}{25}\sin 10\,\omega t+\text{etc.})\ldots\ldots(21)$$

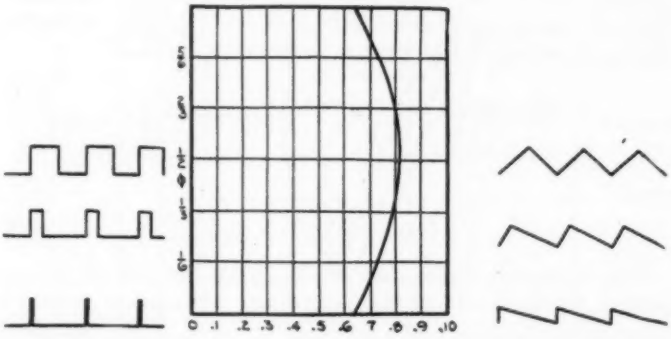

FIG. 2

The form factor for various ratios of rise and fall of a saw-tooth stimulus.

For the saw-tooth stimuli at the extremes of the series, where one arm of the tooth is vertical, the expansion is

$$\frac{\Delta\Theta}{2}\pm\frac{\Delta\Theta}{2}\cdot\frac{2}{\pi}(\sin\omega t+\tfrac{1}{2}\sin 2\,\omega t+\tfrac{1}{3}\sin 3\,\omega t+\ldots.)\ldots.(22)$$

In general, the expression for the complete stimulus is,

$$\Theta=S+\frac{\Delta\Theta}{2}+\frac{\Delta\Theta}{2}\cdot F(\sin\omega t+a_1\sin 2\,\omega t+a_2\sin 3\,\omega t+\ldots.)\ldots(23)$$

where F is a form factor, depending on the ratio of the two arms of the saw-tooth, that is upon Φ. Values of this form factor are plotted in Fig. 2.

The solution of (20) for the boundary condition (23) is

$$\Theta=\left[S+\frac{\Delta\Theta}{2}\right]e^{-x\frac{\mu}{\sqrt{2}}}+\frac{\Delta\Theta}{2}\cdot F\left[e^{-\frac{x}{\sqrt{2K}}\left(\sqrt{\mu^2+\omega^2}+\mu\right)^{1/2}}\left\{\sin\omega t-\frac{x}{\sqrt{2K}}\left[\sqrt{\mu^2+\omega^2}-\mu\right]^{1/2}\right\}\right.$$

$$\left.+a_1e^{-\frac{x}{\sqrt{2K}}\left(\sqrt{\mu^2+4\omega^2}+\mu\right)^{1/2}}\left\{\sin 2\omega t-\frac{x}{\sqrt{2K}}\left[\sqrt{\mu^2+4\omega^2}-\mu\right]^{1/2}\right\}+\text{etc.}\right] \tag{24}$$

Now if the amplitude of this function is small, the part contributed by the periodic terms after the first, involving higher multiples of ω in the exponential term, may be neglected. Discarding these, and putting in the value of $\Delta\Theta$ from (18) we have

$$\Theta=S'+\frac{1}{2c}\,\frac{I\Phi(1-\Phi)}{\omega}\,F\left[e^{-\frac{x}{\sqrt{K}}f(\omega,\,\mu)}\left\{\sin\omega t-\frac{x}{\sqrt{2K}}\left[f'(\omega,\,\mu)\right]\right\}\right]$$

where (25)

$$f(\omega,\,\mu)=\left(\sqrt{\mu^2+\omega^2}+\mu\right)^{1/2} \tag{26}$$

This expression states that we have, by the process of conduction, transformed our sharp saw-tooth stimulus, in general of unsymmetrical shape, into a reaction, (at depth x) of symmetrical sine-curve contour, of much smaller amplitude, the magnitude of the fluctuations dependent both on the amplitude and shape of the saw-tooth stimulus. The three steps from the original flat-topped stimulus, through the photoelectric reaction of unsymmetrical saw-tooth contour, to the finally transmitted symmetrical sine-curve reaction are shown diagrammatically in Fig. 1, *a*, *b*, and *c*.

At this point we must consider the third and last step. What criterion shall we adopt for the visibility of flicker? Several plausible ones suggest themselves, for instance the attainment of a definite range of fluctuation in the transmitted impression; the attainment of a definite fractional range; the attainment of a definite time rate of change of reaction; or any one of these, made to vary in some manner with the magnitude of the illumination

or the sensation.· These three criteria, all perhaps of equal *à priori* probability, have been tried in the present case, with the result that the attainment of a definite rate of change appears to be the only one which, without introducing further complexity, will yield the desired final relations. It is therefore adopted on this strictly pragmatic basis, for which however some additional support is given later. From (25) we get

$$\frac{d\Theta}{dt}=\frac{1}{2c}I\Phi(1-\Phi)Fe^{-\frac{x}{\sqrt{2K}}f(\omega,\,\mu)}\cos(\omega t-\Psi) \quad \ldots\ldots\ldots(27)$$

the maximum value of which is

$$\left(\frac{d\Theta}{dt}\right)_{max}=\frac{I\Phi(1-\Phi)Fe^{-\frac{x}{\sqrt{2K}}f(\omega,\,\mu)}}{2c} \quad \ldots\ldots\ldots(28)$$

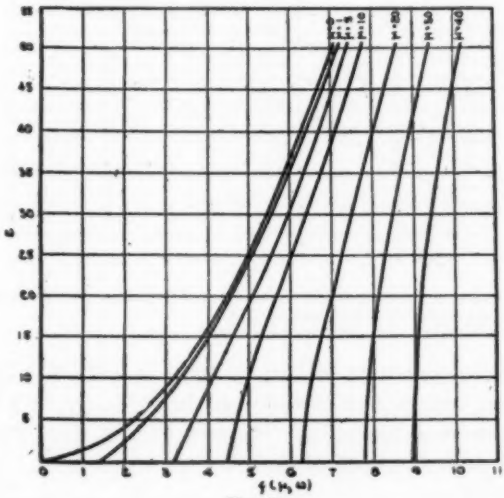

FIG. 3

Values of $f(\omega, \mu)=(\mu+\sqrt{\mu^2+\omega^2})^{\frac{1}{2}}$, in terms of ω.

If our theory is correct we should be able to solve this expression for ω by placing $\left(\frac{d\Theta}{dt}\right)_{max}$ = a constant, and obtain the critical frequency-intensity relations. In order to do this it becomes necessary to investigate the character of $f(\omega, \mu)$ or $(\mu+\sqrt{\mu^2+\omega^2})^{\frac{1}{2}}$. In Fig. 3 are shown calculated values of this function for

various values of μ. It will be seen that for all values of μ above 10, provided ω is greater than 20, $f(\omega, \mu)$ is proportional to ω, so that this function is practically indistinguishable from $(ma+n)$. Substituting this in (28) and solving for ω we get finally

$$\omega = \frac{\sqrt{2K}}{x}\frac{1}{m}\frac{\log I\Phi(1-\Phi)F}{c'} \quad \ldots\ldots\ldots\ldots (29)$$

(where c' is a constant involving $\left(\frac{d\Theta}{dt}\right)_{max}$, c and n) as our general expression for the case of alternating uniformly light and completely dark intervals.

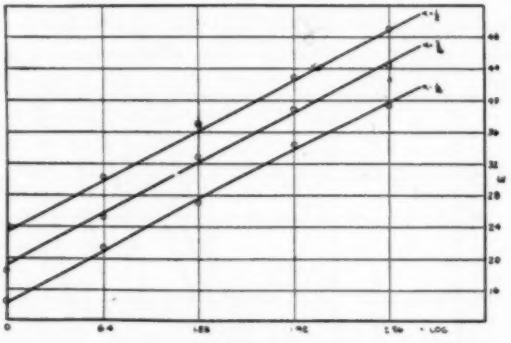

FIG. 4

Critical speeds (ω) versus log illumination for several flicker ranges, sine-curve stimuli.
a = fractional excursion from mean position.
Straight lines drawn to fit equation $\omega = 10 \log Ia + 26.4$.

By similar reasoning to that adopted in deriving (29) we find that if the original stimulus is of equal light and dark intervals, and fluctuates between $\frac{1}{2}+Ia$, and $\frac{1}{2}-Ia$, the amplitude a multiplies into I, giving finally

$$\omega = \frac{\sqrt{2K}}{x}\frac{1}{m}\log\frac{I\,a\,F}{2c'} \quad \ldots\ldots\ldots\ldots\ldots (30)$$

where F has the value corresponding to $\Phi = I - \Phi$, namely, $\frac{8}{\pi^2}$.

These expressions state that critical speeds plot as straight lines against the logarithms of the illuminations; that the speeds for

different amplitudes vary from each other by a factor $\frac{1}{m}\log a$, that the speeds for different openings are represented by a logarithmic function of the ratio of exposure to obscuration. Comparing these findings with the summary of critical frequency relationships symbolized in (1), (2) and (3) it appears that the findings of the theory are in general agreement with the facts. How close this agreement is will be seen from Figs. 4, 5, and 6. In Fig. 4 the

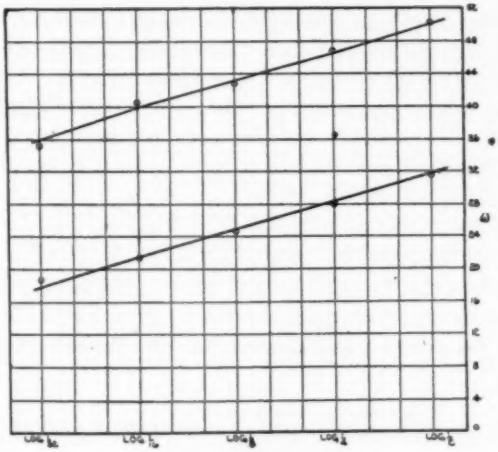

FIG. 5

Critical speeds (ω) versus logarithm of amplitude (a); sine curve stimuli.

circles represent experimental points for a sine curve wave form, the full lines values for various amplitudes calculated from the $a=\frac{1}{2}$ line in accordance with (30), the numerical equation being $\omega=10\log Ia+26.4$, where I is the arbitrary units. (Note that F, while different for a sine-curve stimulus than for a sharp transition one is alike for all amplitudes and so permits the derivation of various amplitude values from a given amplitude irrespective of the wave form of the stimulus). In Fig. 5 where the circles are again new experimental data, the straight line plot of log a against ω is exactly what is called for by (30). We have also for this

case the data of Kennelly and Whiting, who derive empirically the same equation.[14]

As for the speeds at different openings, in Fig. 6 the circles are the writer's 1916 data, the full lines the plot of (29) using the numerical equation $\omega = 10.7 \log IF\Phi(1+\Phi)+30$ derived from the 1916 data; the crosses are new 1921 data. It is evident that the general character of the relationship is well represented by the formula. The writer's experimental data, as already noted, indicate higher speeds for the small openings than for the large, while (29) is symmetrical about $\Phi = \frac{1}{2}$. On the other hand T. C. Porter finds these curves symmetrical. His empirical expression $\omega = a + (b + c \log I)(\log \Phi)(1-\Phi)$ is clearly very like (29). It is to be noted that in the derivation of (29) and (30) no variability with

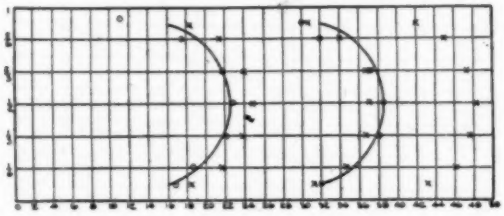

FIG. 6

Critical speeds (ω) versus sector opening (Φ).
Circles, 1916 data. Full lines, calculated curves from equation $a = 10.7 \log I\Phi(1+\Phi)F+30.1$.
Crosses, 1921 data.

intensity has been ascribed to any of the factors. In all probability the diffusivity (K), the rates of recomposition (b and μ) and even the critical rate of change used for the criterion of flicker visibility are functions of the intensity. It is to be remembered as well that the process of derivation of our expressions is approximate only. There would therefore appear to be ample opportunity to account for deviations from the exact relations indicated; the important thing is to account for the main characteristics of the critical frequency relations and this the theory appears to do for the high intensity conditions.

[14] The mutually inclined ω-log I lines obtained by the writer previously for the case shown in Fig. 4 (Phil. Mag., April, 1917, p. 360) are apparently in error, due probably to the short range of intensities available for study in the apparatus then used.

Turning now to the low intensity phenomena, where critical speeds are independent of illumination, the obvious modification demanded if the same theory is to cover this region as well—a simpler one might be found adequate—is some assumption which will result in the "I" term dropping out of (29). Perhaps the simplest assumption is that the process of dark adaptation, which is operative in vision near the threshold, automatically increases the photoelectric sensitiveness (as by supplying more material, or exposing more surface), as the illumination is changed, so as to maintain the mean value of the reaction constant. This adaptation process may be supposed to be altogether too slow to follow the rapid fluctuations of the stimulus which constitute the periodicity to which flicker is due.

This assumption is introduced into the theory by multiplying the right hand side of (13) by the reciprocal of the mean intensity. It will be obvious, without going through the steps, that the various expressions derived for the flicker-wave form relations thereby become independent of the intensity. They reduce in fact to the empirical expression already quoted (4), with the exception that the expression for unequal exposure and obscuration becomes $\omega = \frac{\sqrt{2K}}{x}\frac{1}{m'}\log\frac{(1-\Phi)}{c''}F$, while the empirical expression[5] is $\omega = c \log \frac{\sin \pi \Phi}{\pi \Phi \delta}$. Upon plotting these two expressions however it is found that they are quite indistinguishable in shape. Other expressions, included in the general empirical form, and applying to sine curve and other wave forms, are not handled by the present approximate treatment, but it is highly probable that an exact solution of our general equation (as altered to cover the low intensity condition) would yield results agreeing equally well with the empirical expression found to fit the experimental data. It may be pointed out that the observation of a lower limit to flicker speed in the low intensity investigation is exactly in accord with the criterion of a definite rate of change of transmitted impression as the critical condition for perception of flicker. At low speeds the transmitted impression rises and falls slowly because of the slowness of change

of the stimulus; at high speeds it rises and falls slowly because of the smoothing out processes at work due to conduction. These two limits of flicker speed may be considered as support for the adoption of the rate of change criterion.

ELECTRICAL MODEL ILLUSTRATING THEORY

In the theory as stated there is nothing which absolutely binds us to an electrical mechanism, probable although that may be. The initial reaction may be characterized simply as photochemical, the conduction process may be a diffusion of decomposition products, the criterion of visibility of flicker may be rate of change of concentration of these products. The mathematical treatment is general and is the same for electrical as for chemical processes and either may figure at some or all stages. It is, however, of some interest, as contributing to concreteness, to illustrate the theory by an electrical model which is governed by the equations used. In Fig. 7 let S be a photoelectric cell of the ordinary vacuum type. At L let there be a leak, of high resistance which

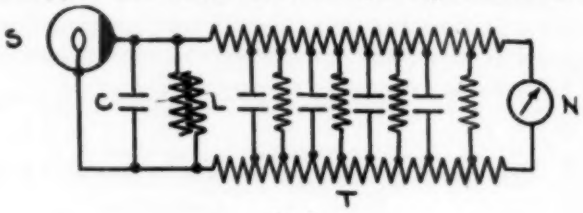

FIG. 7

Electrical model illustrating theory.
S photoelectric cell
C condenser
L variable resistance leak
T "Cable" with distributed capacity and leakage
N sensitive receiver.

decreases as the applied voltage increases, in such a manner that the attained potential is approximately as the logarithm of the stimulus (certain loose contacts approximate this property). At C, close to the cell, let there be a capacity. The rest of the system T consists of a "cable" consisting of a resistance path, with distributed capacity and leakage,—the leakage in the cable being composed of ordinary non-varying resistances. At the far end of

the line is to be placed a detecting instrument N which starts to indicate when the rate of change of potential across the two arms of the cable reaches a certain critical value. The *amplitude* of an *induced* current would be a criterion corresponding to that postulated.

The two varieties of resistance leaks assumed for ease of description, can be reduced to one, the variable resistance kind,—since it would only be the resistances close to the cell which would be subjected to voltages high enough to utilize the departture from Ohmic character. Also the capacity pictured near the cell may be merely the normal capacity of the cable near the cell. The whole system may, therefore, be physically somewhat simpler than the coupling of photoelectric cell, capacity, leak, and special transmitting channel which must be considered as separate entities for purposes of mathematical treatment. It is, in fact, quite possible that all the recombination and diffusion properties required may be localized in the liquid photoelectric cell itself.[15]

NUMERICAL FORMULAS

The values of the constants to be used with the formulas above derived depend upon the illumination unit, the size of the observing field, the particular observer. Porter's formula for equal dark and light intervals has been rather widely copied; it appeared therefore worth while to calculate the constants for the new formulae to agree with his.

His formula for high intensities is

$$\omega = 12.4 \log I + 29.4 \quad \ldots\ldots (31)$$

where ω is in cycles per second and I in meter candles. [His slope, (12.4) is higher than Kennelly and Whitings (11) and that fitting Fig. 4, (10)]. Observing that according to our notation $I\alpha$ must be substituted for I, α being $\frac{1}{2}$, we get *for equal light and dark exposures*, of amplitude α

$$\omega = 12.4 \log I\,\alpha + 33.1 \quad \ldots\ldots (32)$$

For various openings (Φ), for $\alpha = \frac{1}{2}$, we get similarly

$$\omega = 12.4 \log I\,\Phi\,(1-\Phi)F + 38 \quad \ldots\ldots (33)$$

[15] For the influence of diffusion on the response of a liquid photoelectric cell, see Samsonow, Zeits, f. Wiss. Phot. XI, 1912, p. 33.

where F is the fraction shown in Fig. 2. Making use of the observation that $\Phi(1-\Phi)F$ is practically equivalent to

$$\frac{\sin \pi \Phi}{\pi \Phi} \times \text{const.}$$

we get a handier working formula

$$\omega = 12.4 \log I \frac{\sin \pi \Phi}{\pi \Phi} + 35.6 \quad \ldots\ldots\ldots\ldots (34)$$

For *low intensities* (blue light), using the writer's own observations,[5] for *various amplitudes*

$$\omega = 13.3 \log a + 18.6 \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

for *various openings*

$$\omega = 13.3 \log (1-\Phi)F + 21 \quad \ldots\ldots\ldots\ldots\ldots (36)$$

or noting that $(1-\Phi)F$ is practically equivalent to $\frac{\sin \pi \Phi}{\pi \Phi} \times$ const. we get this working formula:

$$\omega = 13.3 \log \frac{\sin \pi \Phi}{\pi \Phi} + 17.2 \quad \ldots\ldots\ldots\ldots (37)$$

With these formulas a complete family of low and high intensity critical frequency curves, for abrupt transitions of illumination, may be plotted, which represent the experimental data closely in character and position.

DISCUSSION

The most serious objection to the theory of intermittent vision here presented appears to the writer to be the fact that the logarithmic relation between illumination and critical frequency is due to the special type of conduction assumed for the products of the light action. It would appear *à priori* much more likely that this is a more or less direct consequence of the logarithmic relation between stimulus and response. In that respect the lines of thought in the attempted theoretical derivations of Kennelly and Troland are preferable.

It may also be felt, owing to the somewhat lengthy mathematical development, that the processes assumed are unduly complex, and that some other simpler method of handling the variables at our command based on different assumptions might give the same results. It should be emphasized that the complexity is due to

the mathematical processes themselves, and that the assumed physical processes—an initial photoelectric or photochemical reaction, and a subsequent conduction—are simple and plausible. On the basis of these assumed physical processes the mathematical treatment cannot be much simpler, whatever direction it takes. Such ultimate explanation of the critical frequency phenomena as may be developed will unquestionably involve processes of reaction to light and transmission of the results of the reaction. The theory here presented should therefore be at least a guide to a more complete (and probably even more complex) theory which can be built up on a better knowledge than we now possess of photochemical reactions and of physiological conduction processes.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND THE
WESTERN ELECTRIC COMPANY, INCORPORATED.
JULY, 1921.

THE VISIBILITY FUNCTION AND VISIBILITY THRESHOLDS FOR COLOR-DEFECTIVES

BY
MARGARET C. SHIELDS

Information with regard to the luminosity sense of color-defective eyes has seemed to the writer, interested because she is herself a deuteranope, meagre and contradictory. No adequate effort has been made to connect the type of color deficiency with the form of the luminosity curve, and the question of absolute luminosity has been, except for the work of Sir William Abney, untouched. The writer was recently accorded the privilege, as a guest of the Nela Research Laboratory, of obtaining her own visibility curve on the special equality-of-brightness spectrophotometer developed in that laboratory, and desires here to present the results of that work, together with a few observations upon similar data already on record.

The apparatus is essentially a Lummer-Brodhun spectrophotometer, the field of the ordinary size, and the collimators arranged so that brightness matches are made step by step through the spectrum for almost indistinguishable color differences. The writer acknowledges appreciatively her indebtedness for the use of assembled and calibrated apparatus, and also for the determination of the brightness temperatures of the sources, from which the energy distribution was computed. Both the apparatus and the experimental procedure were exactly as originally described.[1]

There are on record carefully determined visibility functions for sixteen persons recognized to be color defective. Watson[2] has offered five cases, with the thesis that on the three process theory as represented by the Abney curves an observer, in proportion as he fails to get the green stimulus from white, should require more white to match in brightness a red than a normal observer, the

[1] Astrophys. J., *35*, p. 237, 1912; and *48*, p. 65, 1918.
[2] Proc. Roy. Soc., *88*, p. 404.

effect being to shift his maximum visibility to the red; conversely, an observer who loses relatively more red than green, should have his maximum visibility shifted to the green. He failed, however, to demonstrate for his three cases which are shifted to the red, that they were actually found by other tests to be more

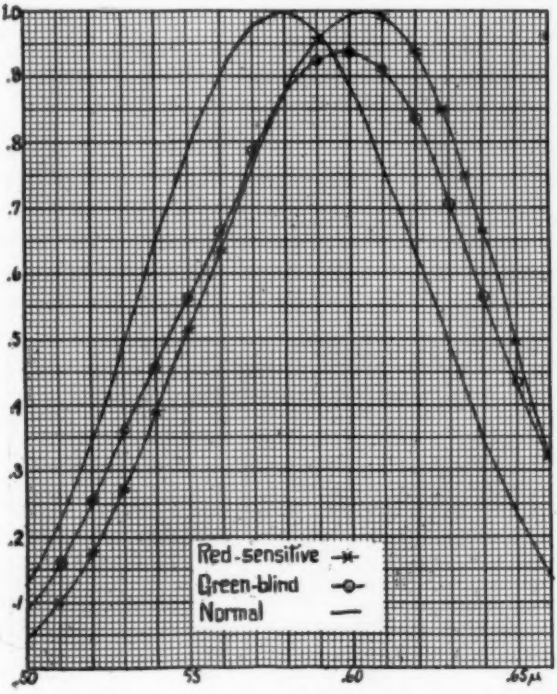

FIG. 1
Equal-area luminosity curves (Source at 2045°K).

blind to green than red, and for the two shifted to the green vice versa. Tufts[3] presents three cases of color-defectives showing a shift to the red, and three to the green, with no attempt to correlate these shifts with the type of defect. Coblentz[4] finding

[3] Phys. Rev., *25*, p. 433.
[4] Bul. Bur. Std. *14*, p. 167.

four cases of shift to the red and one to the green, specifies that two of the former were green-blind by the Nagel test, but makes no statement as to the other three.

The finding of the writer in her own case adds one more to the two cases of Coblentz which definitely conform with Watson's

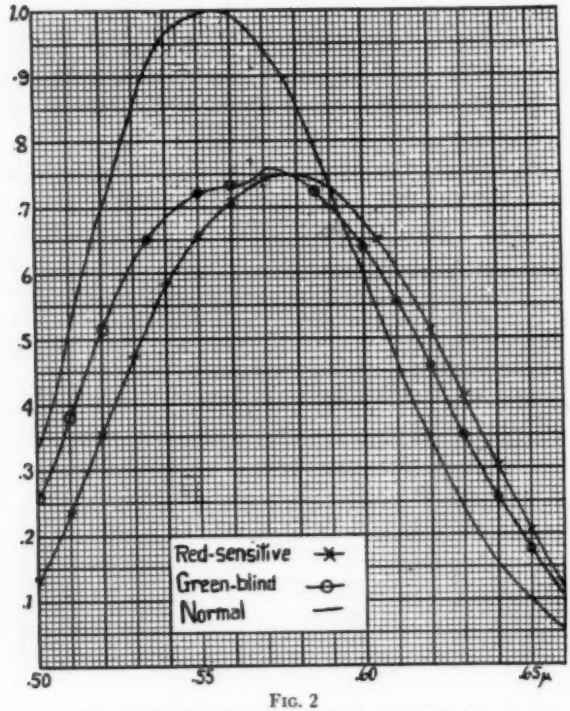

FIG. 2
Visibility functions derived from the luminosity curves of Fig. 1.

theory. She is a deuteranope, failing completely any perception of green, but undoubtedly seeing the quality red except when the saturation is fairly low,[5] and her visibility curve has its maximum at 571 $m\mu$, as compared with 556–7 $m\mu$ for the normal eye. (See the green-blind subject of Fig. 2.) It seems to the writer unfortu-

[5] See Hayes, Am. Jour. Psych., *22*, p. 369, case M.S. This opinion is also supported by Prof. Dunlap of Johns Hopkins.

nate that all these visibility studies were not accompanied by adequate quantitative tests of the color sense of the subjects, but that it is none the less fairly probable that relative blindness to red must somehow predicate a shift of the maximum visibility to the green, and vice versa.

There is this quantitative difference between the point of view of Watson and the measured cases, that the totally green-blind should have their maximum definitely in the red, (the Abney curve, assuming the temperature of the arc to be 3500°C, would bring the maximum at 590$m\mu$) whereas the largest displacement found is one of Coblentz' cases at 578$m\mu$. Moreover, contrary to the statement of Watson, it is to be noted that the integral luminosity of the green-blind may be arbitrarily made equal to the normal without giving rise to an abnormally high maximum luminosity in the red. (See Fig. 1.) This is true for the present case and for all four of Coblentz' cases having the maximum in the red. The form of the curve is indeed closely comparable with that of a normal red-sensitive subject, as shown in Fig. 1. The integral visibility derived from equal-area luminosity curves is of course below normal for any curve with its maximum in the red, but it is to be remembered that these curves represent luminosity for an equal energy spectrum, not the conditions of actual vision.

Further interest attaches to the correspondence of the present case with previous observations in that all the latter were taken by a flicker method in which the observer measured the brightness of the color in terms of white as he saw it, whereas this curve was taken by an equality-of-brightness photometer using red at 650 $m\mu$ as a standard, since each wave length was measured in terms of a shorter, starting at this point. Ives and others have raised the question as to whether the two methods measure precisely the same thing, and have suggested that in flicker the cones are relatively more important. In such case, if color-blindness be at the same time rightly ascribed to a deficiency of the cones, the recognized divergence between the two methods might be found much more striking for color-defective eyes. The qualitative agreement between the two methods may therefore be taken as indicating that the two are none the less as nearly equivalent for

color-blind as for normal eyes, or that the observed shifts are not attributable to the method.

On the basis of the simple three process theory which makes the total luminosity sense at each point in the spectrum the sum of the ordinates of three color sensation curves, Abney was led to draw

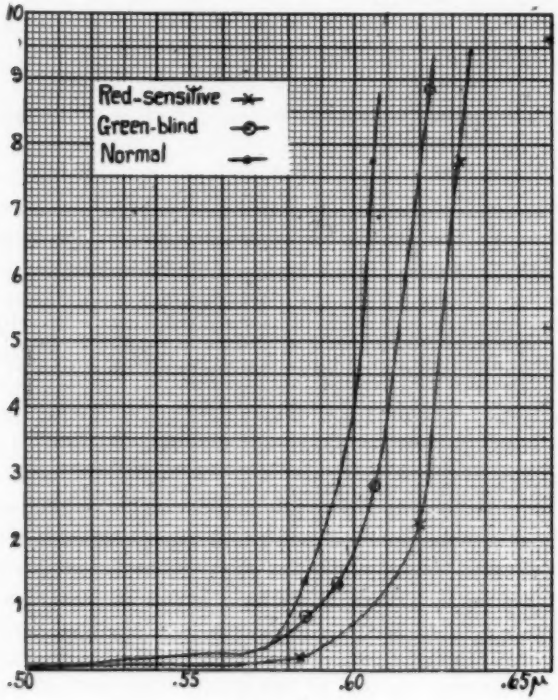

FIG. 3
Visibility thresholds.

curves giving for a person totally lacking the green sense an integral luminosity only .7 the normal, and for a red-blind person only .3. This extreme lowering of the absolute brightness sensibility of color-defectives he justified partly on the basis of threshold determinations. It was therefore a matter of considerable interest to test this by comparing the visibility thresholds for a

few persons whose visibility functions had been determined in the original investigation in the Nela Laboratory with that of the one available color-blind case. This was accomplished with the same spectrophotometer, using only one lamp, operating it at a low temperature, narrowing the slit, then allowing the observer

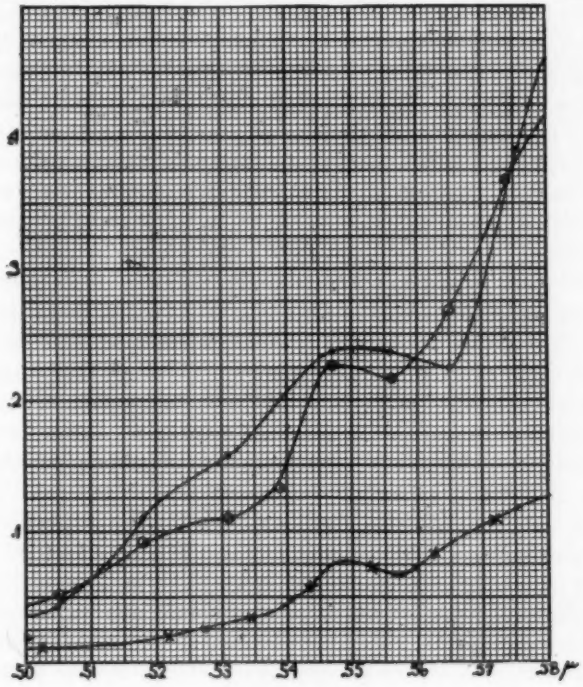

FIG. 4
Detail of Fig. 3.

to set the rotating disc so as just to extinguish the pattern, his eyes being in a stable state of dark adaptation. The energy distribution was determined as before from the equivalent black body temperature of the filament, 1375°K, with the correction applied for dispersion. The results are given for three typical cases in Fig. 3, showing in arbitrary units the minimum energy perceptible as a function of the wave length; Fig. 4 shows the

detail of the short wave length portion of the same curves. Judgments of what constitutes extinction are difficult to make with precision, but it is at least certain that in the case of the writer there is no evidence of marked loss of brightness sensibility. As compared with this one deuteranope, the red-sensitive subject of Figs. 1 and 2 with normal color vision has a somewhat lower threshold throughout; the third, a subject, whose visibility is close to the average has one somewhat higher throughout. Still another normal observer required the slit width to be doubled before he could see the pattern at all, obtaining then a curve nearly identical in form with that of the green-blind subject. The three threshold curves lie in the red in the inverse order to the corresponding visibility curves, but in the same order in the blue, so that they cannot be correlated absolutely. It would of course be desirable to measure the least discernable brightness difference for these subjects from the threshold up to ordinary levels; but even without this, if, contrary to Abney's assertion, a green-blind may actually perceive a smaller minimum of energy than a normal, not only in the spectral region where the relative visibility is high but beyond the green where his relative visibility is low, there is the possibility that his integral luminosity at ordinary brightness levels may not be markedly below normal.

The statement made by Tufts and by Coblentz is amply justified, that an abnormal visibility function is not necessarily associated with color-defective vision; but it is certainly equally true that there is no case on record of a color-defective with a normal visibility function. The existing evidence indicates, rather, that color-defective vision does condition a perfectly definite modification of the visibility function; it would therefore appear that a theory of vision should interrelate brightness sense and color sense to the extent of accounting for this. It seems doubtful, however, if color-blindness does involve a lowered brightness sense in the extreme fashion in which the Young-Helmholtz theory has been interpreted to involve it.

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SOUTH HADLEY, MASS.

RECALESCENCE IN ANTIMONY

BY
ENOCH KARRER, PHYSICIST

A common illustration of recalescence is found in iron. When an iron wire cools from an incandescent temperature, it will suddenly become brighter when the recalescent point is reached. A far more striking case of such recalescence is found in antimony. Many substances recalesce, but in order that the recalescence shall be easily seen with the unaided eye it must occur at or above a temperature giving a just sensible brightness. In general, for any given change of temperature brought about by the molecular rearrangement during recalescence the relative change in brightness is greater the lower the temperature. From this standpoint antimony offers advantages, since recalescence in it takes place at or below the melting point (630°C.). This temperature is sufficiently higher than the minimum temperature[1] to make the luminous mass of metal easily visible. The usual method of detecting recalescence is in cooling curves.[2] Foote[3] has already called attention to this phenomenon in antimony as used in pyrometric calibration. However, when the effect is observed indirectly, as by heating an enclosure, it cannot be so marked as when the surface of the metal itself is observed.

The recalescence in antimony is very striking when a globule of the metal is heated in a glass tube to a temperature above 630°C. Either the surface of the globule in contact with the glass or the opposite surface may be viewed. To make the measurements recorded in the curves of Figs. 2 and 3, a disposition of apparatus shown in Fig. 1 was satisfactory. The antimony globule was placed in a small furnace and in contact with a small thin-walled quartz tube into which a thermocouple (Pt-Pt Ir) was inserted.

[1] A black body subtending an angle of perhaps 1 radian is still luminous at a temperature of 530°C to a dark adapted eye.

[2] i.e., temperature vs. time.

[3] Met. and Chem. Eng., *11*, p. 97, 1913.

The globule is also covered over with a thin piece of quartz plate. A photometer was placed a short distance above the surface of the antimony. Two observers[4] read respectively the galvanometer connected with the thermocouple[5] and the ammeter of the photometer,[6] while a third observer matched the photometric fields as the metal cooled. The data given in the curves of Figs. 2 and 3 are the averages obtained from many samples of antimony. The

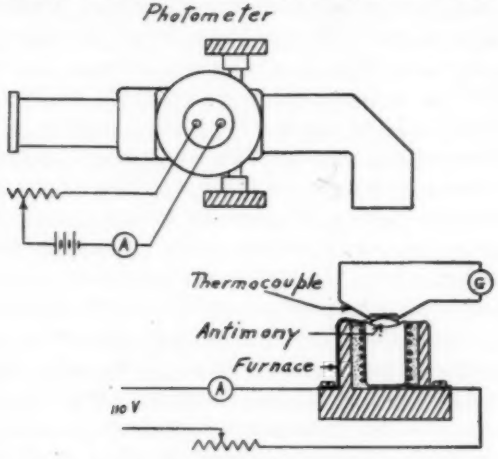


FIG. 1

Disposition of apparatus to observe the recalescence in antimony.

first results plotted in Fig. 2 were obtained in a very shallow furnace allowing the globule to cool very quickly. The recalescence occurs as nearly as can be determined at the melting point

[4] I am indebted to Mr. R. H. Sinden and Mr. T. I. Ángell for much assistance in these observations.

[5] The thermocouple was calibrated by observing the deflection of the galvanometer when globules of equal sizes of antimony, lead and tin were in succession melted in the furnace. The calibration curve is a straight line whose intercepts depended upon the size of the globule and the position of the quartz tube and junction, and therefore changed with each sample. The calibration curve however could easily be adjusted for every new charge of the furnace by observing the melting point of the antimony under test.

[6] The photometer was calibrated to give a relation between brightness in millilamberts (candles per $cm^2 = \pi$ lamberts) and the current through the photometer lamp. A red glass was used for color matching.

of the metal. The flashing was of rather too short duration, (20 to 35 seconds) for ease of observation. A new furnace was constructed slightly deeper than the first in order that the cooling may be less rapid (about 55 seconds) and in order that the temperature may more easily be controlled. Fig. 3 shows the results plotted, again the mean of a large number of observations. In this case the recalescence takes place far below the melting point. In fact in some cases the globule was no longer visible before the flashing occurred. The metal was likewise in a solid phase in such cases.

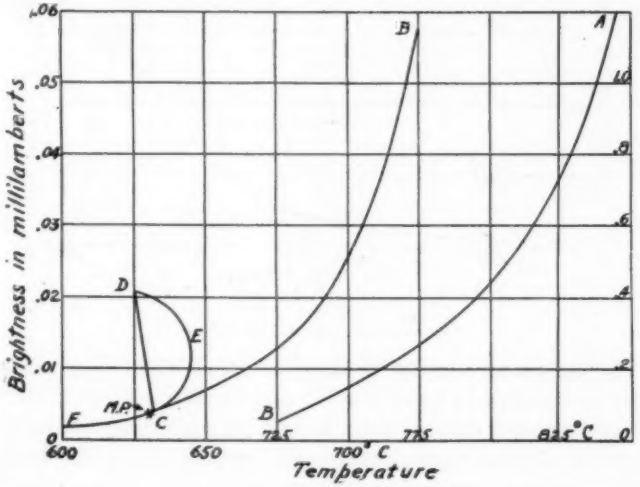


FIG. 2

Temperature-brightness curve of antimony.

In many instances the antimony was heated up to a temperature above 700°C. The brightness-temperature relation is given by the portion AB of the curve (Fig. 3). The cooling proceeds along this curve to point F where the average of all observations seems to indicate a slight retardation in the rate of decrease of the brightness until the flashing point C is reached where the brightness suddenly jumps to a high value, D, and remains so for an appreciable time while the temperature is rising. The tempera-

ture to which the metal rises always remains below the melting temperature. It is almost impossible to obtain a sufficient number of observations by ordinary photometry to determine accurately what the shape of the loop *FCDEF*, may be. On the return to the point *F* the brightness-temperature curve becomes continuous with the first (*AB*) portion of the curve.

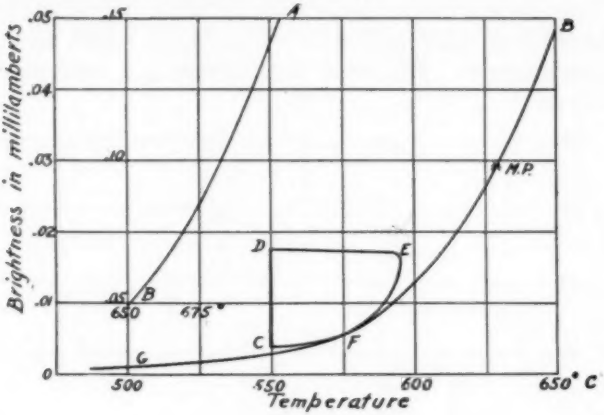


FIG. 3

Temperature-brightness curve of antimony.

The temperature may be kept at some value intermediate between the melting point (630°C) and the flashing point (580°C) for an extended time without in any way affecting the flashing when the ultimate cooling lowers the temperature to the flashing point.[7] Such a temperature was maintained on one occasion for about one minute and on another occasion for eight minutes.[8] In the latter case the uncertainty in the temperature equilibrium due to the lag in the thermocouple is eliminated. During this time the metal is solid. To test this point the cover was removed and the globule pricked with a small rod. The fact that the metal is solid before the flashing means that this case of recalescence is

[7] I am indebted here to Mr. R. H. Sinden for carrying out several experiments to check this and other points.

[8] It may be noted however that in one single case where the time was more than ten minutes no flashing was observed in subsequent cooling.

exactly analogous to that in iron and that it is not merely an under cooling of the liquid metal as has been thought heretofore.

This phenomenon was incidently observed while antimony and other metals were melted to make alloys for thermocouples. None of the other metals (tin, bismuth, cadmium) exhibit it. In one impure sample of bismuth it was noticed. Antimony is detectable by the recalescent phenomenon in bismuth when in such small proportions as 1:2000.

The recalescence takes place in an atmosphere of hydrogen and in vacuum in the same manner as in air.

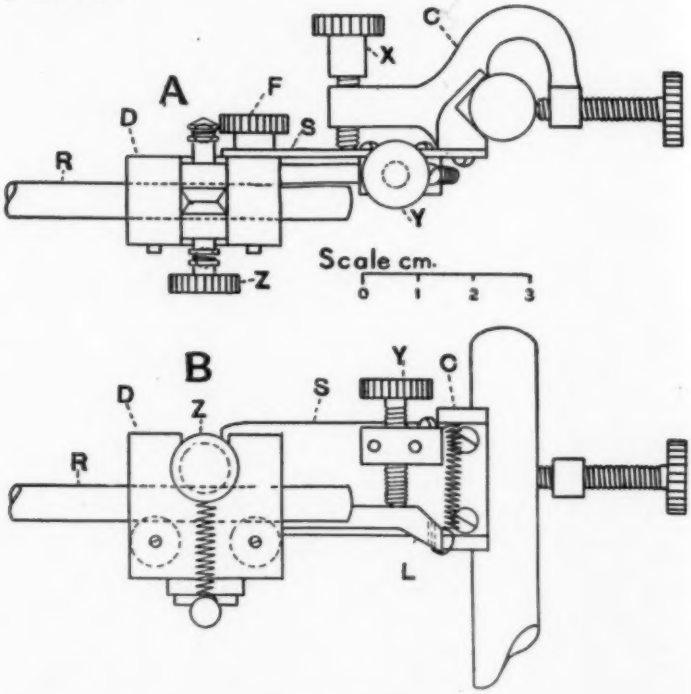
LABORATORY OF APPLIED SCIENCE,
NELA RESEARCH LABORATORIES,
NELA PARK, CLEVELAND, OHIO, JANUARY 20, 1922.

INSTRUMENT SECTION

THREE-PLANE ORIENTATION CLAMP

BY
W. R. MILES

A small lens, writing point, microscopic slide or other such object may be adjusted freely and accurately in the three planes of geometric space by use of this special clamp. A top view is shown in diagram *A* and a side view in *B*. The device is composed of two principal parts, *C* and *D*, both of which are "slow motion" clamps regularly sold by well known instrument makers. It is the ***combination*** of the parts which is thought to be original and not previously described.



The part designated *C* provides for attachment to a suitable supporting stand and has an extension in the form of a leaf spring, *S*, which is acted upon by the adjusting screw, *X*. This provides for motion in one plane since *S* is rigidly attached to *C* at one end while the other carries part *D*. A rod, *R*, bearing the object to be adjusted passes between the three grooved rollers of *D* as seen in diagram *B*. The upper roller with knurled head *Z* is under tension from coil springs which act to pull it against *R*. The turning of *Z* causes *R* to move right or left and the movement is positive and delicate. The third plane of adjustment is about *F* as a fulcrum for *D*. The short lever, *L*, which is an extension of the frame of *D*, is held in contact with the screw, *Y*, by a small coil spring. The turning of *Y* thus causes the left hand end of *R* (sketch *B*) to be raised or lowered as desired. All three motions are free from backlash, are practically independent of each other, and as shown by the diagrams have considerable ranges of orientation.

NUTRITION LABORATORY
BOSTON, MASS.

NOTE ON A METHOD OF INCREASING THE CARRYING CAPACITY OF A RHEOSTAT

BY
W. E. FORSYTHE

When using an ordinary slide resistance to control a current it often happens that when the current is the largest, only a small part of the resistance is being used. This may be a disadvantage for two reasons: in the first place the smallest change possible in this small part of the resistance may greatly increase the current and in the second place the resistance may be very much overheated.

A method has been devised whereby it is possible to lessen both these difficulties by using both ends of the rheostat and thereby double its carrying capacity and at the same time make it much easier to control the current.

The working of the rheostat is best explained by a reference to the figure. In Fig. 1 the resistance is shown, by the heavy lines with the two main binding posts of the rheostat at *A* and *B*. In this case the rheostat is made up with sixteen steps. As such a rheostat is ordinarily used, the current enters at *A* passes through

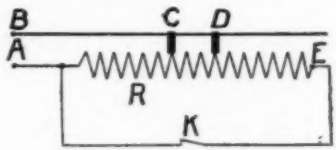


FIG. 1

the resistance *R*, then through some sort of a sliding contact, shown at *C*, and out through the binding post *B*. To increase the current, the sliding contact *C* is moved forward, thus short circuiting one or more turns of the rheostat. With the rheostat shown there are sixteen steps in the variation of the current. For a rheostat where each step is equal, as more and more of the steps are cut out a single step becomes a greater percentage of what

remains. For this reason the current control becomes very poor as the current is increased. There is another disadvantage in such a rheostat and that is this—when the current is increased by cutting out more and more of the resistance there is apt to be too large a current through the part of the resistance being used while a part of the rheostat is not in use at all.

To increase the carrying capacity of such a rheostat the additional sliding contact D is added and also another binding post E at the opposite end of the rheostat from the binding post A. The binding posts A and E are connected through the switch K so that by closing the switch K the two ends of the rheostat can be connected.

To operate the rheostat in this form proceed in the ordinary manner (switch K open) with the current entering at A passing through the resistance R, the sliding contact C and out at B. To increase the current move the sliding contact C towards A until it has passed over three-fourths of the distance from E to A, that is, until the resistance being used is one-fourth that of the entire rheostat. Now to obtain a larger current move both sliding contacts C and D to the center of the rheostat R and close the switch K. The resistance of the rheostat with switch K closed and the sliding contacts in this position is equal to about one-fourth of the total resistance of the rheostat, that is, the resistance will be about the same as before the sliding contact C was moved back to the center. The current will divide and one-half will enter at A and pass through one end of the rheostat and out through the sliding contact C while the other half will enter at E, pass through the other end of the rheostat and out through the sliding contact D. To increase the current move either the sliding contact C towards A or move the sliding contact D towards E or move both the sliding contacts C and D towards the ends A and E.

If a better control of the current is wanted, increase the current to about the desired value by moving but one sliding contact and then make the final adjustments by moving the other sliding contact in whatever direction necessary. By this method a very much better control of the current is possible and at the same time the carrying capacity of the rheostat is greatly increased.

Even for a rheostat when the resistance of the steps towards the end becomes less and less this method of use with two sliding contacts is very advantageous. In this case care will have to be taken not to send the same current through the two ends of the rheostat. However a very much better control will be obtained. Two rheostats connected as described above have been in use in this laboratory for the past three or four years and have been very satisfactory. One is a water cooled nichrome-ribbon rheostat intended to carry about forty to fifty amperes. When fitted with two sliding contacts and connected up as described above it will carry about eighty to one hundred amperes and at the same time it is possible to get good adjustment of the current.

It is quite obvious that this method can be carried a step farther by addition of two more sliding contacts and at the same time add a binding post at the center of the rheostat so that the center can be connected through another switch K' to the two ends A and E. In this case when the two sliding contacts C and D are brought to the center of the rheostat the other two sliding contacts G and H are to be moved to the center. The sliding contacts G and H are to be between C and D. To operate move but two of the sliding contacts C and D away from the center until each has passed over three-quarters of the length of that half of the resistance. Now move two sliding contacts to within one quarter of the distance to each end of the rheostat and close the switch K'. The resistance being used will be about the same as it was when the two sliding contacts were each within one-eighth of the distance from each end. To further increase the current move C and D towards opposite ends of the rheostat or move G and H towards the center. Since each sliding contact can be moved alone or in connection with any one or more of the others, it can be seen that this method with four sliding contacts will give a very good control.

NELA RESEARCH LABORATORIES,
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CONSTRUCTIONAL DATA FOR A CEMENTED OBJECTIVE OF BARIUM CROWN AND FLINT

BY
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The smaller optical fire control instruments used in the Army, viewed from the optical standpoint, consist essentially of a low power telescopic system designed to be as compact as is consistent with a large exit pupil. This in turn calls for a cemented objective of short focal length corrected for a large relative aperture. In view of this, it has been thought advisable to compute the following tables which have been obtained by graphic interpolation from a selected series of objectives which were carefully computed and checked by means of trigonometric ray tracing.

The usual type of construction has been assumed in which the crown component is in front. Standard thicknesses of .04 and .02 f have been adopted for the crown and flint components, these relatively large values being necessary to provide sufficient thickness at the edge for a short focus objective of large aperture. Furthermore, these thicknesses lead to economical production since ample margin is provided for repolishing if scratches should develop in the final finishing. Barium crown 1.570 to 1.580 and flint 1.610 to 1.622 have been selected as the glasses upon which the tables are based while the dispersion and indices for the *C* and *F* spectrum lines have been obtained by a smoothing process from sets of average values, determined from a large number of melts furnished the government during the war. For this reason, the specifications for the glass in these tables can be satisfied, almost identically, by glass of American manufacture readily obtainable. This range of glass includes those which were shown by Harting[1] to be most favorable for simultaneously eliminating spherical and chromatic aberration and satisfying the sine condition.

The arrangement of the tables is self evident. Table 1 gives the optical constants of the glass. Two barium crowns, numbers 1 and

[1] Harting, Zeitschrift für Instrumentenkunde, Vol. 18 p. 355 (1898).

2, and a single series of flint glasses are listed. Tables 2, 3, and 4 give directly the first, second and third radii for a lens of unit focal length corrected for spherical and chromatic aberration in such a manner that the edge ray for an aperture of f/5 and the paraxial rays for the *D* spectrum line have the same back focal length and in addition the paraxial rays for the *C* and *F* spectrum lines have the same equivalent focal lengths. The desired radius is found in each table at the intersection of the vertical column corresponding to the crown and the horizontal column corresponding to the flint glass. Four significant figures have been retained. This has not required an extravagant amount of extra labor and it has much facilitated the checking of the tables by a consideration of the differences. It is not meant to imply that the reproduction of radii to this accuracy is necessary in order to obtain satisfactory lenses.

Table 5 gives the values of the departure from the sine condition for the lenses listed. In all cases the lenses are undercorrected as regards departure from the sine condition, i.e., the height of the incident ray divided by the sine of the final slope angle is less for a ray distant from the center than for a paraxial ray.

TABLE 5

Departure from Sine Conditions for F/5 Doublet[4] Corrected for Spherical and Chromatic Aberration

Indices	1.570	1.572	1.574	1.575	1.576	1.578	1.580
1.610	.0051	.0049	.0047	.0046	.0045	.0042	.0040
1.612	.0051	.0049	.0047	.0046	.0044	.0042	.0040
1.615	.0051	.0048	.0046	.0045	.0044	.0043	.0041
1.617	.0050	.0048	.0046	.0045	.0044	.0043	.0042
1.620	.0049	.0048	.0046	.0045	.0045	.0043	.0042
1.622	.0048	.0047	.0046	.0045	.0045	.0043	.0042

[4] In the above table there is tabulated, in terms of f, the difference between the equivalent focal length of D rays entering at the axis and the value of $h/\sin a'$ for a ray incident at a point distant $f/10$ from the axis. In all cases there is undercorrection, that is, the equivalent focal length of the $f/10$ ray is less than that for the axial ray.

TABLE 1

Constants of Glass upon Which Tables are Based

N_C	N_D	N_F	$N_F - N_C$	
BARIUM CROWN, NUMBER 1				
1.56706	1.570	1.57708	.01002	56.88
1.56805	1.571	1.57811	.01006	56.77
1.56904	1.572	1.57913	.01009	56.66
1.57002	1.573	1.58015	.01013	56.55
1.57101	1.574	1.58118	.01017	56.44
1.57199	1.575	1.58220	.01021	56.33
1.57298	1.576	1.58323	.01025	56.22
1.57397	1.577	1.58425	.01028	56.11
1.57496	1.578	1.58528	.01032	56.00
1.57595	1.579	1.58631	.01036	55.90
1.57694	1.580	1.58734	.01040	55.80
BARIUM CROWN, NUMBER 2				
1.56720	1.570	1.57703	.00983	57.99
1.56818	1.571	1.57805	.00987	57.85
1.56916	1.572	1.57907	.00991	57.72
1.57015	1.573	1.58010	.00995	57.59
1.57114	1.574	1.58113	.00999	57.46
1.57213	1.575	1.58216	.01003	57.33
1.57311	1.576	1.58318	.01007	57.20
1.57410	1.577	1.58421	.01011	57.07
1.57509	1.578	1.58523	.01014	57.00
1.57608	1.579	1.58626	.01018	56.88
1.57707	1.580	1.58729	.01022	56.75
MEDIUM FLINT				
1.60541	1.610	1.62174	.01633	37.35
1.60638	1.611	1.62278	.01640	37.25
1.60735	1.612	1.62382	.01647	37.15
1.60832	1.613	1.62487	.01655	37.05
1.60930	1.614	1.62592	.01662	36.94
1.61027	1.615	1.62697	.01670	36.84
1.61124	1.616	1.62802	.01678	36.74
1.61221	1.617	1.62906	.01685	36.64
1.61318	1.618	1.63010	.01692	36.54
1.61415	1.619	1.63115	.01700	36.43
1.61512	1.620	1.63219	.01707	36.33
1.61609	1.621	1.63324	.01715	36.23
1.61706	1.622	1.63429	.01723	36.13

TABLE 2

First Radius of an F/5 Doublet[1] Corrected for Spherical and Chromatic Aberration

Indices	1.570	1.571	1.572	1.573	1.574	1.575	1.576	1.577	1.578	1.579	1.580
1.610	.4656	.4681	.4707	.4733	.4761	.4790	.4820	.4851	.4882	.4914	.4946
1.611	.4660	.4685	.4710	.4736	.4764	.4792	.4820	.4849	.4878	.4908	.4938
1.612	.4664	.4689	.4713	.4739	.4766	.4793	.4820	.4847	.4875	.4903	.4930
1.613	.4669	.4693	.4717	.4742	.4768	.4794	.4820	.4846	.4872	.4898	.4923
1.614	.4674	.4697	.4721	.4745	.4770	.4795	.4820	.4845	.4869	.4894	.4917
1.615	.4679	.4702	.4725	.4749	.4773	.4797	.4821	.4844	.4867	.4890	.4912
1.616	.4684	.4707	.4729	.4752	.4776	.4799	.4821	.4844	.4866	.4887	.4908
1.617	.4690	.4712	.4734	.4756	.4779	.4801	.4822	.4844	.4865	.4885	.4905
1.618	.4696	.4718	.4739	.4760	.4782	.4803	.4823	.4844	.4864	.4884	.4903
1.619	.4703	.4724	.4744	.4764	.4785	.4805	.4825	.4844	.4864	.4883	.4902
1.620	.4711	.4730	.4749	.4768	.4788	.4807	.4826	.4845	.4864	.4883	.4902
1.621	.4719	.4737	.4755	.4773	.4791	.4810	.4828	.4846	.4865	.4883	.4902
1.622	.4727	.4744	.4761	.4778	.4795	.4813	.4830	.4848	.4866	.4884	.4902

[1] Barium Crown, Number 1, is to be employed in this series of lenses. The crown component faces the object (assumed at an infinite distance). In all cases the first radius is positive, i.e., convex towards the object.

TABLE 3

Second Radius of an F/5 Doublet[a] Corrected for Spherical and Chromatic Aberration

Indices	1.570	1.571	1.572	1.573	1.574	1.575	1.576	1.577	1.578	1.579	1.580
1.610	.3248	.3228	.3207	.3186	.3164	.3142	.3119	.3095	.3070	.3044	.3017
1.611	.3279	.3259	.3238	.3217	.3195	.3173	.3150	.3127	.3102	.3077	.3052
1.612	.3309	.3289	.3268	.3247	.3226	.3204	.3181	.3158	.3134	.3109	.3083
1.613	.3340	.3318	.3297	.3277	.3256	.3234	.3211	.3188	.3165	.3141	.3117
1.614	.3370	.3348	.3327	.3307	.3286	.3264	.3242	.3219	.3196	.3173	.3150
1.615	.3400	.3378	.3357	.3337	.3316	.3294	.3272	.3249	.3227	.3204	.3182
1.616	.3429	.3408	.3387	.3367	.3346	.3324	.3302	.3280	.3258	.3236	.3213
1.617	.3458	.3437	.3416	.3396	.3375	.3354	.3332	.3311	.3289	.3267	.3245
1.618	.3486	.3465	.3445	.3425	.3405	.3384	.3362	.3341	.3320	.3298	.3276
1.619	.3513	.3493	.3473	.3454	.3434	.3413	.3391	.3370	.3350	.3329	.3307
1.620	.3540	.3520	.3501	.3483	.3463	.3442	.3421	.3400	.3381	.3360	.3338
1.621	.3567	.3548	.3530	.3512	.3493	.3472	.3452	.3431	.3411	.3390	.3368
1.622	.3593	.3576	.3559	.3541	.3523	.3503	.3483	.3463	.3442	.3420	.3397

[a] Barium Crown, Number 1, is to be employed in this series of lens. The second radius is negative in all cases, i.e., concave towards the object. Thickness of crown component .04 f.

TABLE 4

Third Radius of an F/5 Doublet[4] Corrected for Spherical and Chromatic Abberration

Indices	1.570	1.571	1.572	1.573	1.574	1.575	1.576	1.577	1.578	1.579	1.580
1.610	5.654	5.772	5.902	6.046	6.207	6.385	6.583	6.799	7.034	7.300	7.611
1.611	5.766	5.893	6.029	6.168	6.341	6.521	6.717	6.930	7.159	7.404	7.676
1.612	5.897	6.033	6.178	6.333	6.500	6.677	6.867	7.069	7.284	7.511	7.747
1.613	6.054	6.194	6.345	6.505	6.675	6.853	7.037	7.226	7.420	7.618	7.818
1.614	6.242	6.396	6.555	6.717	6.882	7.048	7.215	7.384	7.554	7.725	7.895
1.615	6.467	6.632	6.794	6.953	7.109	7.261	7.411	7.558	7.701	7.843	7.982
1.616	6.734	6.913	7.076	7.223	7.361	7.491	7.616	7.737	7.855	7.970	8.082
1.617	7.044	7.216	7.364	7.499	7.622	7.735	7.840	7.938	8.031	8.120	8.207
1.618	7.395	7.551	7.685	7.801	7.904	7.997	8.083	8.162	8.234	8.301	8.366
1.619	7.785	7.917	8.027	8.122	8.204	8.277	8.342	8.403	8.460	8.514	8.567
1.620	8.208	8.298	8.378	8.449	8.513	8.571	8.624	8.674	8.721	8.766	8.810
1.621	8.649	8.698	8.745	8.792	8.838	8.882	8.925	8.967	9.009	9.050	9.090
1.622	9.082	9.102	9.124	9.150	9.178	9.209	9.244	9.281	9.320	9.360	9.400

[4] Barium Crown, Number 1 is to be employed in this series of lens. The third radius is positive in all cases, i.e., convex towards the object. Thickness of flint component .02 f.

TABLE 6

First Radius of an F/5 Double[a] Corrected for Spherical Aberration and Sine Condition

Indices	1.570	1.571	1.572	1.573	1.574	1.575	1.576	1.577	1.578	1.579	1.580
1.610	.6340	.6347	.6354	.6360	.6365	.6371	.6378	.6386	.6394	.6402	.6412
1.611	.6346	.6352	.6359	.6365	.6371	.6378	.6385	.6392	.6400	.6408	.6417
1.612	.6352	.6358	.6365	.6371	.6378	.6385	.6392	.6399	.6406	.6414	.6422
1.613	.6358	.6364	.6370	.6377	.6384	.6391	.6398	.6405	.6412	.6419	.6426
1.614	.6363	.6369	.6376	.6383	.6390	.6397	.6404	.6410	.6417	.6424	.6430
1.615	.6368	.6374	.6381	.6388	.6395	.6402	.6409	.6415	.6421	.6428	.6434
1.616	.6372	.6378	.6385	.6392	.6399	.6406	.6413	.6419	.6425	.6431	.6437
1.617	.6376	.6382	.6389	.6395	.6402	.6408	.6415	.6421	.6426	.6432	.6438
1.618	.6379	.6385	.6391	.6397	.6404	.6410	.6416	.6422	.6427	.6433	.6438
1.619	.6382	.6387	.6393	.6399	.6405	.6410	.6416	.6422	.6427	.6432	.6437
1.620	.6384	.6389	.6394	.6399	.6404	.6409	.6414	.6420	.6425	.6430	.6435
1.621	.6385	.6389	.6393	.6397	.6401	.6406	.6411	.6416	.6421	.6425	.6430
1.622	.6384	.6387	.6390	.6393	.6397	.6400	.6404	.6409	.6414	.6418	.6423

[a] The crown component faces the object (assumed at an infinite distance). In all cases the first radius is positive, i.e., convex towards the object.

TABLE 7

Second Radius of an F/5 Doublet[a] Corrected for Spherical Aberration and Sine Condition

Indices	1.570	1.571	1.572	1.573	1.574	1.575	1.576	1.577	1.578	1.579	1.580
1.610	.3254	.3234	.3209	.3186	.3164	.3141	.3119	.3096	.3073	.3050	.3028
1.611	.3279	.3260	.3237	.3214	.3192	.3169	.3147	.3124	.3101	.3078	.3055
1.612	.3305	.3284	.3263	.3242	.3220	.3197	.3175	.3152	.3129	.3106	.3082
1.613	.3329	.3310	.3290	.3269	.3247	.3225	.3203	.3180	.3158	.3134	.3109
1.614	.3354	.3337	.3317	.3296	.3274	.3252	.3230	.3208	.3186	.3162	.3136
1.615	.3379	.3362	.3343	.3323	.3302	.3280	.3258	.3236	.3214	.3190	.3163
1.616	.3404	.3387	.3368	.3349	.3329	.3307	.3284	.3263	.3241	.3217	.3191
1.617	.3428	.3411	.3393	.3374	.3354	.3333	.3310	.3289	.3268	.3245	.3219
1.618	.3451	.3434	.3417	.3399	.3379	.3358	.3336	.3315	.3294	.3272	.3247
1.619	.3473	.3456	.3440	.3422	.3403	.3382	.3361	.3340	.3320	.3298	.3274
1.620	.3495	.3478	.3461	.3443	.3425	.3405	.3385	.3365	.3345	.3323	.3300
1.621	.3516	.3498	.3481	.3463	.3445	.3427	.3408	.3388	.3369	.3348	.3326
1.622	.3535	.3518	.3500	.3483	.3465	.3447	.3429	.3410	.3391	.3372	.3352

[a] The second radius is negative in all cases, i.e., concave towards the object. Thickness of crown component .04 *f*.

TABLE 8

Third Radius of an F/5 Doublet[1] Corrected for Spherical Aberration and Sine Condition

Indices	1.570	1.571	1.572	1.573	1.574	1.575	1.576	1.577	1.578	1.579	1.580
1.610	2.663	2.698	2.733	2.768	2.806	2.845	2.884	2.924	2.965	3.013	3.065
1.611	2.636	2.670	2.704	2.738	2.775	2.813	2.851	2.890	2.930	2.975	3.023
1.612	2.610	2.643	2.676	2.710	2.745	2.781	2.818	2.857	2.897	2.939	2.982
1.613	2.584	2.616	2.649	2.683	2.717	2.751	2.788	2.825	2.864	2.905	2.945
1.614	2.559	2.591	2.623	2.655	2.689	2.723	2.759	2.796	2.834	2.872	2.911
1.615	2.535	2.566	2.598	2.630	2.663	2.696	2.731	2.767	2.804	2.842	2.880
1.616	2.512	2.542	2.574	2.605	2.637	2.670	2.704	2.739	2.776	2.813	2.851
1.617	2.490	2.520	2.551	2.581	2.613	2.646	2.679	2.714	2.750	2.787	2.826
1.618	2.469	2.499	2.529	2.560	2.591	2.623	2.656	2.690	2.726	2.764	2.803
1.619	2.450	2.479	2.509	2.539	2.570	2.602	2.635	2.668	2.703	2.741	2.783
1.620	2.432	2.461	2.491	2.520	2.550	2.582	2.615	2.649	2.685	2.722	2.765
1.621	2.416	2.445	2.474	2.503	2.532	2.564	2.598	2.633	2.669	2.707	2.749
1.622	2.402	2.430	2.459	2.488	2.518	2.550	2.583	2.619	2.655	2.694	2.734

[1] The third radius is negative in all cases, i.e., concave towards the object. Thickness of flint component .02 f.

TABLE 9

Axial Chromatic Aberration for F/5 Doublet[a] Corrected for Spherical Aberration and Sine Condition

WHEN BARIUM CROWN, NO. 1, IS EMPLOYED

Indices	1.570	1.572	1.574	1.576	1.578	1.580
1.610	.0028	.0027	.0026	.0025	.0024	.0022
1.616	.0028	.0027	.0027	.0026	.0025	.0023
1.620	.0028	.0027	.0026	.0026	.0025	.0024
1.622	.0027	.0027	.0026	.0026	.0025	.0024

WHEN BARIUM CROWN, NO. 2, IS EMPLOYED

1.610	.0019	.0019	.0018	.0016	.0015	.0014
1.616	.0020	.0019	.0019	.0018	.0016	.0015
1.620	.0019	.0019	.0018	.0018	.0017	.0016
1.622	.0019	.0018	.0018	.0018	.0017	.0016

[a] In the above table there is tabulated, in terms of f, the difference between the equivalent focal length for the F and C rays near the axis. In all cases there is undercorrection, i.e., the equivalent focal length for the F ray is less than that for the C ray.

Tables 6, 7, and 8 give radii of a second series of lenses and parallels tables 2, 3, and 4 respectively with the difference that spherical aberration is eliminated and the sine condition satisfied for paraxial rays and the edge rays for a zone of aperture f/5. Table 9 gives the outstanding axial chromatic aberration of the equivalent focal length for this second series of lenses. Two sets of values are given, the one applying when the barium crown, No. 1, is utilized, the other when the No. 2 is used. In all cases there is under correction, that is, the equivalent focal length for the F is less than for the D spectrum line.

The accuracy of the computations is such that it is believed the aberrations which have been "eliminated" in each case will be found to be less than .003 f over the greater part of the table and well under .005 f in all parts.

Fig. 1 shows in the usual manner the spherical aberration and departure from sine condition for a typical lens selected from the center of tables 2, 3, and 4. Fig. 2 shows similar curves for the lenses of Tables 6, 7 and 8. Fig. 3 shows the curvature of field and

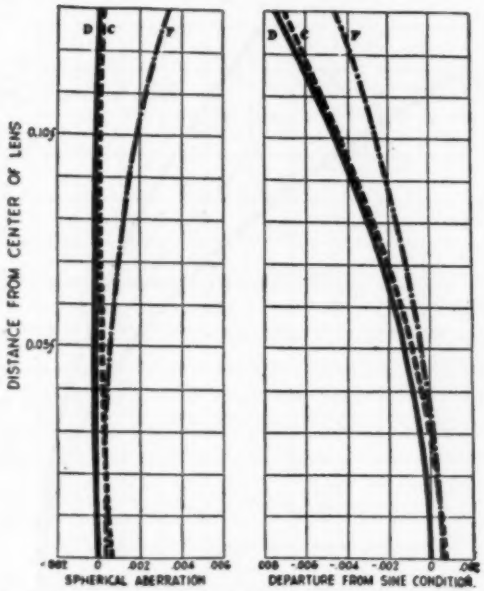

FIG. 1—Aberrations of lens corrected for spherical and chromatic aberration.

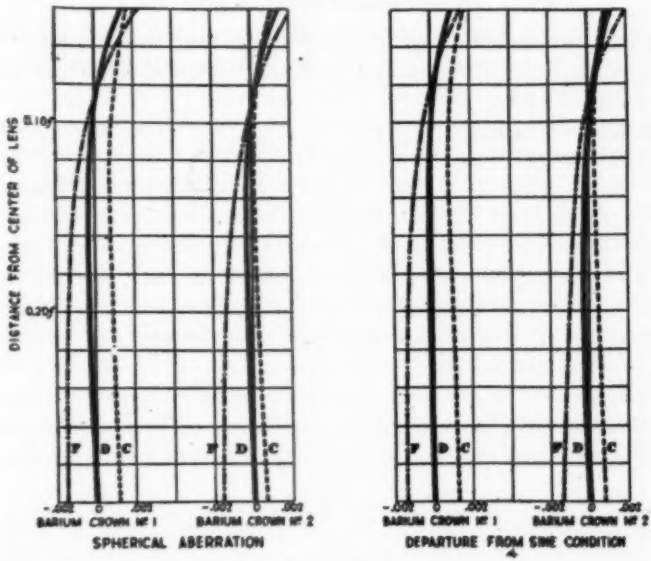

FIG. 2—Aberrations of lens corrected for spherical aberration and departure from the sine condition.

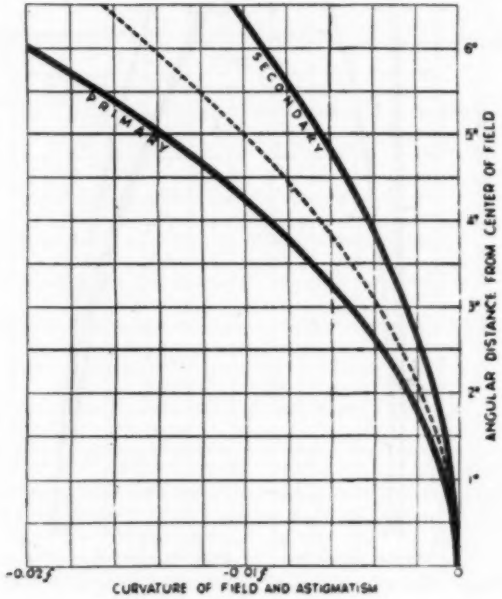

FIG. 3—Curvature of field and astigmatism.

astigmatism and may be considered as applying to either series of lenses since these aberrations are practically identical for the two sets of tables.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

AN APPARATUS FOR STUDYING THE MOTION OF RELAYS

BY
HERBERT E. IVES
AND
T. L. DOWEY

In the study of the electrical phenomena accompanying the making and breaking of contacts by relays, it is of considerable importance to know definitely how the contact points move relatively to each other. The electrical methods which have been commonly employed for determining relay characteristics are quite inadequate for the kind of study here in mind. Oscillographic records of current tell merely whether contact is single of multiple, without any direct indication of the nature of the motions which result in contact. Methods for measuring the velocity of relay arms depending on time determination by the charging of condensers are only applicable to clean makes or breaks and so are impracticable for the study of the very important case of "chattering."

The apparatus which is illustrated in Figures 1 and 2 was designed for the purpose of obtaining photographic silhouettes, or "shadowgrams" of the moving contact points. The optical arrangement, while presenting no essentially new features, is one which is made eminently practical by the availability of an extended light source of high brightness, in the "pointolite" lamp. By reference to Fig. 1 the manner of its use will be made clear. First of all an enlarged image of the incandescent pointolite ball is thrown by means of a lens into the plane of the relay contacts. Then an image of the contacts and the pointolite image is formed by a second lens upon a narrow slit, parallel to the direction of motion of the contacts, and lying closely in front of a rapidly moving photographic film. It is clear that the shadow traced on the film will be a record of the relay's motion. The remaining problem is one of mechanical and electrical design to secure the proper control of film speed, the

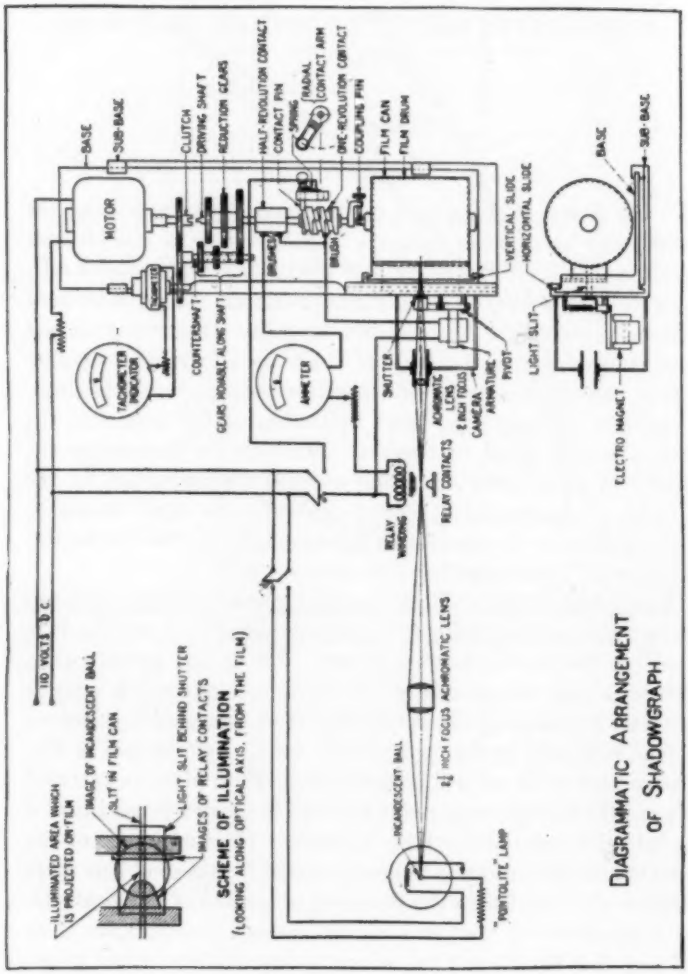

FIG. 1
Diagrammatic sketch of shadowgraph and connections.

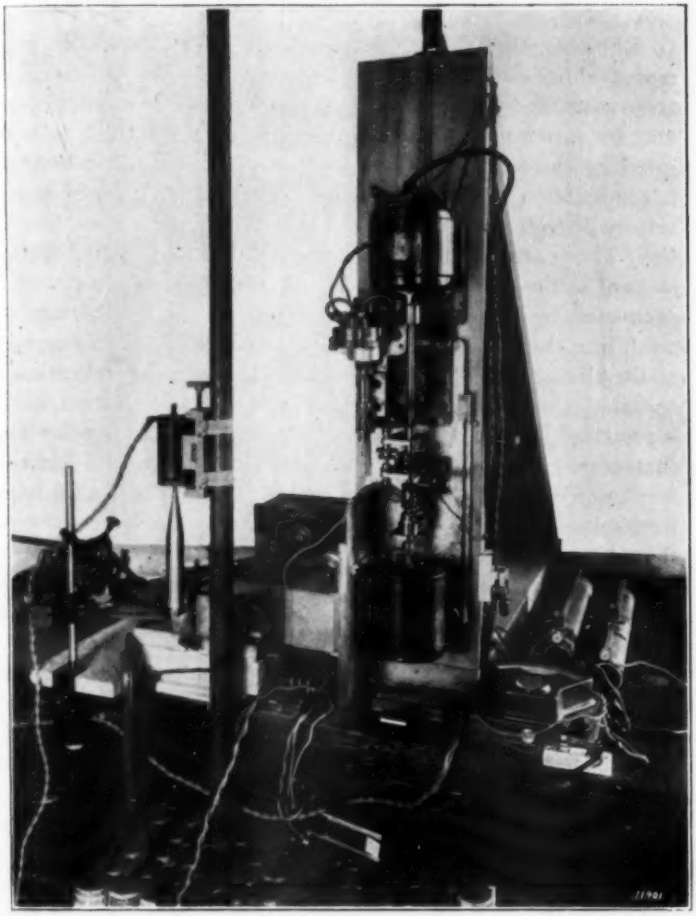

FIG. 2

Shadowgraph mounted in vertical position for photographing opening characteristics of relay operated by falling weight.

starting and stopping and proper synchronism of the various parts, and arrangements to facilitate the various operations involved.

As shown diagrammatically in Fig. 1 the film drum (a standard G. E. oscillograph part) is driven by a 110 volt 1/10 H.P. D.C. motor, either directly, or through trains of gears. By means of these gears, and by the insertion of series resistance the film speed may be varied from 15 to 1200 centimeters per second, as indicated by the electric tachometer mounted on the counter-shaft in combination with the known gear ratios. It is of course necessary to provide that the film be exposed only during one revolution. This is arranged by placing a magnetically controlled shutter in front of the film slit, which is only opened when the circuit is completed by dropping the pin carried on the "radial contact arm" into the helically grooved one revolution contact mounted on the driving shaft. Where the relay to be measured is electrically operated it may be controlled by a half revolution contact which is mounted on the same shaft. Proper timing of the relay and shutter may be effected by the angular setting of the full and half revolution contacts on the driving shaft. The film can and driving mechanism are arranged to slide parallel to the slit so that several shadowgrams may be made on the same film. The entire apparatus may be set up either horizontally or vertically (as shown in Fig. 2) depending on the mode of operation of the moving parts it is desired to photograph.

The performance of the apparatus is well shown by the selection of shadowgrams Figs. 3 and 4. The first set (Fig. 3) shows a series of opening characteristics all made in succession upon a single film of a lever relay operated by a falling weight. Here the zero opening is indicated by a strip of finite width because the shadow actually cast is that of two small knife edge "ears" carried by the contact pieces. The magnification of the relay opening here used was about five diameters, which is ample for measuring purposes and at the same time leaves the illumination quite adequate even with a very narrow slit (.1 mm) for the highest speed of rotation of the film can. Fig. 4 shows various cases of "chatter," in commercial relays with one or both relay arms vibrating. Simple inspection of these shadowgrams gives valuable information on

FIG. 3

Opening characteristics of falling weight operated relay for various heights of fall, as photographed on single film. (Long period vibrations at top of each record are caused by quivering of the pointolite ball image.)

FIG. 4

Shadowgrams of poorly adjusted commercial relays showing various kinds of "chatter."

the performance of the relays photographed, while accurate measurements of the films open the way for quantitative analysis of the motions.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY, AND THE
WESTERN ELECTRIC COMPANY, INCORPORATED.
MARCH 24, 1922.

INFRA RED TELEGRAPHY AND TELEPHONY

BY
T. W. CASE

During 1916–1917, a search was made for new light reactive materials which would change their electrical resistance on exposure to light. The list of materials examined has been published in the Physical Review.[1] One of the compounds in this list, lead antimony sulphide, was found to be somewhat active to the infra red. It was also noted that most of the new light reactive compounds found were sulphides. Therefore, further investigation was carried on having to do mainly with compounds containing sulphur. The most promising of these compounds was found to be a thallium sulphide. This substance showed a light action mainly due to the infra red.

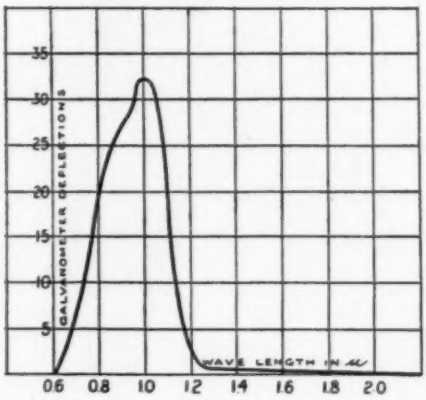

FIG. 1

Fig. 1 shows the spectral action of this thallium sulphide as measured by Dr. W. W. Coblentz.[2] The sensitivity of the material

[1] Physical Review, Vol. IV, No. 4, April, 1917.
[2] Bureau of Standards paper No. 380.

was greatly increased by a method of preparation in which the thallium sulphide was slightly oxidized and then placed in a vacuum.

Fig. 2 shows the finished cell which has been termed the "Thalofide Cell." The slightly oxidized thallium sulphide is coated upon a quartz disc, the disc having been previously heated to melt the material which is placed on it. Conducting lines of graphite are drawn over the coated surface and the element mounted in a glass tube, which is then evacuated.[3]

FIG. 2

The action of this cell is in a great many respects similar to the well known selenium cell except for the fact that its action is mainly due to the longer rays as shown in the spectral curve. It also differs in that the induction and deduction periods are more rapid than those of selenium. The resistance of the cells, (5 to 500 megohms) is very much higher than that of the ordinary selenium cell. This latter condition was sought because of the

[3] U. S. Patent number 1316350.

attendant high (per cent) sensitivity attained between light and dark exposure. This makes the cells ideal for work in conjunction with audion bulbs where low resistance cells are at a disadvantage.

As this cell was developed with the idea of using it for infra red signaling, it was necessary to find an efficient light filter which would transmit the infra red, but not the visible rays. It was found that when Wratten filters, numbers 91, 45, and 53, were superposed, no visible light was transmitted and a high percentage of the infra red came through. This datum was sent to Dr. C. E. K. Mees of the Eastman Kodak Company's Research Laboratory with a request that he combine these three filters into one, which he was successful in doing. The gelatin film was coated on optical plate glass, and the film protected by a suitable cover glass. Ordinary plate glass or window glass, which shows a green color at the edge, was found unsuitable for the transmission of the infra red around 1 micron. The whole filter unit was sealed at the edges to prevent moisture from attacking the gelatin. The transmission of the infra red filter as finally developed is shown in Fig. 3. This curve was plotted by Dr. W. W. Coblentz.

To have an efficient infra red signaling system, it seemed desirable that the signals detected by the Thalofide cell should be made audible. To accomplish this, an audion arrangement was used. The method of interrupting the light at the sending end at an audible frequency was first tried and found to be impractical. Next, it was decided to interrupt the current through the Thalofide cell at the receiving end by mechanical means, for an audible frequency. Dot and dash signals would then be heard as changes in intensity of the audio frequency through the Thalofide cell. These could then be amplified by an audion bulb. This method was discarded a short time later, when it was found that certain connections with an audion bulb containing gas at a prearranged pressure could be made to produce an oscillating current through the Thalofide cell. In this latter arrangement, when the Thalofide cell changed in resistance, the pitch of the audio frequency current changed. This was found to be a very much more sensitive method of registering small changes of resistance in the Thalofide cell.

The audion bulbs used were of the Western Electric V type. To make these bulbs sensitive for use they were connected to a vacuum system, and baked, and exhausted to a very high vaccum.

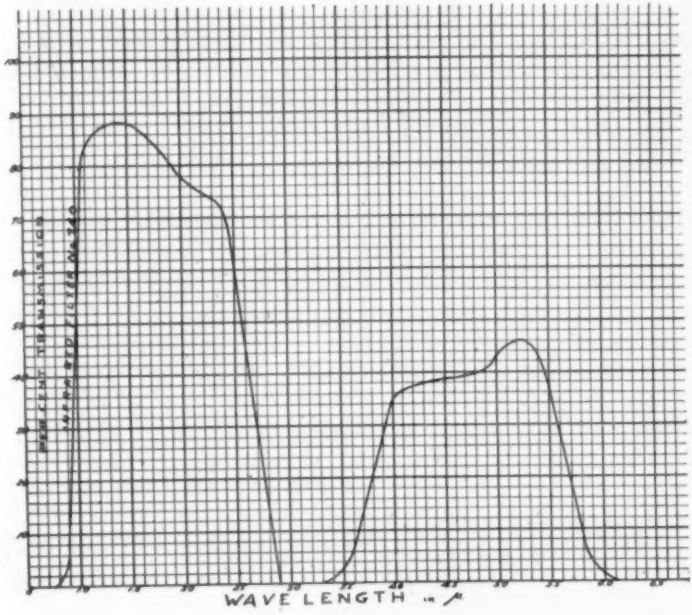

FIG. 3

Argon was then let in and the pressure adjusted until the bulbs oscillated freely at minimum voltage on a test circuit with a predetermined resistance in place of the Thalofide cell.[4] The wiring used is shown in Fig. 4.

The phone potential is fixed at 120 volts. The cell potential is variable and totals 200 volts. This is to take care of the different dark resistances of the cells. In use, the audion filament is brought to a dull red and the voltage adjusted to give a note of the required pitch in the phones (usually about 1000 cycles) after which the pitch may be changed by slightly adjusting the filament rheostat.

[4] U. S. Patent number 1379166.

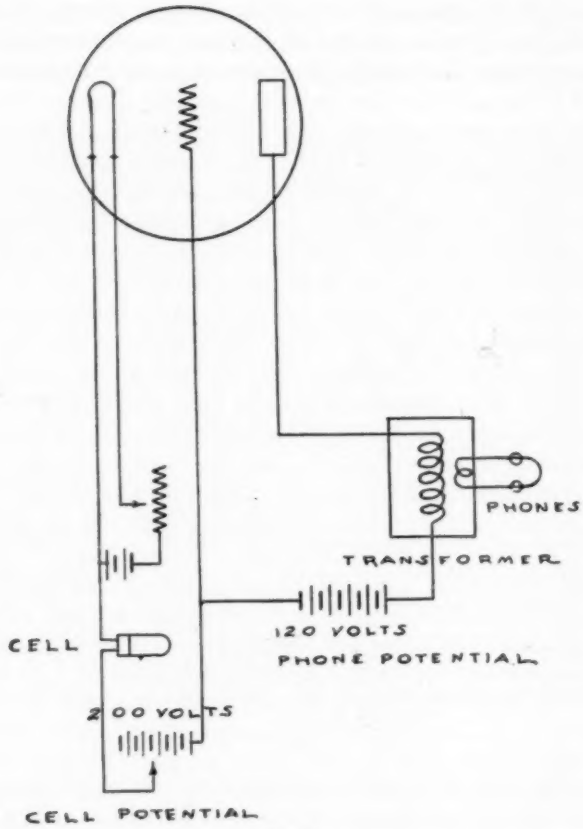

FIG. 4

The action of the Thalofide cell in this circuit may be likened to a valve through which a potential builds up on the plate until a discharge occurs through the gas of the bulb. A fraction of a second is then required until the potential builds up again through the cell. The time rate of building up of consecutive charges and discharges may be changed by increasing or decreasing the filament current with its consequent effect in the ionization of the gas. The sequency of charge and discharge causes the audio

frequency note heard continually in the phones. When the resistance of the Thalofide cell is lowered by a received signal, the time rate of charge and discharge is varied which is made evident by a change of pitch of the audio frequency note in the phones.

In actual signal work, the Thalofide cell is placed at the focus of a parabolic mirror which serves as a collector of the infra red beam. For a projector of the beam any suitable search light may be used which is properly screened with an infra red filter. A light beam is then projected which is invisible to the eye, and which may be detected by the above described receiving apparatus.

Some of the first official tests of this apparatus were made before representatives of the army and navy in October 1917 between Fort Hancock and the Woolworth building, a distance of about eighteen miles. Messages were successfully transmitted. The signals were so loud that they could be heard at some distance from the phones. These results were obtained in spite of the usual smoky atmospheric conditions between these two points.

The sending apparatus consisted of a Sperry 60 inch search light situated at Fort Hancock and screened by an infra red filter made for this light.

The receiving apparatus consisted of a 24 inch parabolic mirror with the Thalofide cell at its focus. The Thalofide cell was connected to the oscillating circuit described above.

Further tests were made in February, 1918 in conjunction with the coast artillery at Fort Monroe, Va.

At the end of the war, a four mile land set was constructed. This consisted of two eight inch drums, one for sending and one for receiving, rigidly mounted with their axes parallel. The sending projector was mounted at the top. It was made up of an eight inch parabolic reflector with a special eight volt signal lamp placed at the focus. Between the lamp and reflector, a small butterfly shutter was mounted. This could be operated by a key at the side of the apparatus, by means of which the signals could be sent. An infra red filter was placed over the front of the drum. The lower drum was used as the receiver. It contained an eight inch parabolic mirror and a Thalofide cell mounted at the focus of the

mirror. The Thalofide cell was connected, by means of a flexible lead, to the receiving box which contained the audion circuit. Power was furnished for the lamps and audion by a small storage battery.

The reason that the sending and receiving drum were mounted with parallel axes was to render it easy for two operators to pick each other up in the dark. If one operator sweeps the horizon with his receiving drum and hears a flash signal from the other operator, he leaves his apparatus at the point where he hears the flash signal. He then turns on the invisible beam in the sending drum. His invisible beam will now cover the point from which he received his signal and by sending dots the other operator locates him in like manner.

This portable set can be easily carried and operated by two men and has a range of about four miles.

The atmospheric transmission of the infra red as detected by the Thalofide cell and galvanometer, when compared with the visible as indicated by photometric readings taken with a Sharp-Millar photometer, is shown in the curves, Fig. 5.

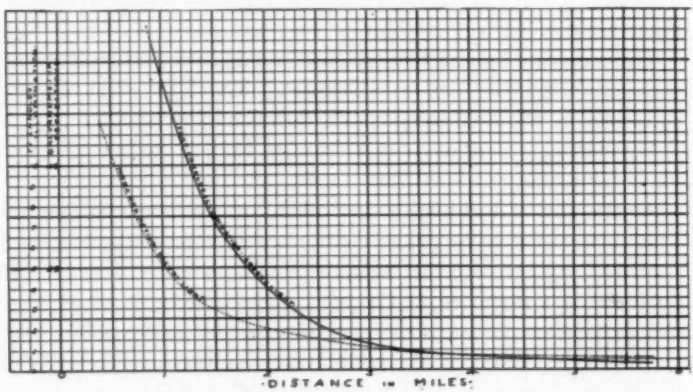

FIG. 5

These curves were taken on a fairly clear night and show that the infra red rays are transmitted through the atmosphere with less absorption than the visible rays.

During this time, work was also being done to adapt this system for actual telephony with the infra red. The first problem was the development of a suitable and constant high intensity light source, which could be easily varied by sound. The result of this research was a modification of the ordinary manometric flame principle, consisting of a special form of an oxy-acetylene burner in which the oxygen is directed obliquely against opposite sides of an acetylene flame. (See Fig. 6.) This makes a small intense light source, having an area of about .25 square inches and a candle-power of about one hundred and fifty.

FIG. 6

For the receiving end, the Thalofide cell was connected to a straight audion hook-up for simple and direct amplification. On a clear night, using twenty-four inch sending and receiving projectors, with three step amplification the voice could be easily

understood at five miles. Similar small sets were made up, consisting of twelve inch projectors and receivers, which gave very good voice tranmission at two miles.

It was realized, at this time, that there would be no demand for such apparatus or its further development except for war purposes, consequently it was decided to stop all further work along this line.

CASE RESEARCH LABORATORY,
AUBURN, N. Y.

NOTICES

The Optical Society of America

MINUTES OF SIXTH MEETING

Helmholtz Memorial Meeting

The sixth meeting of the Optical Society of America was held in Rochester, N. Y., October 24, 25, 26, 1921. One hundred thirteen persons were registered in attendance. The attendance at various sessions varied from about 35 to 100 or more.

The most notable feature of the meeting was the Helmholtz Memorial Meeting held on the afternoon and evening of Monday, October 24. The following former students of Helmholtz were present: Prof. Henry Crew, Prof. C. R. Mann, Prof. Ernest Merritt, Prof. E. L. Nichols, Prof. M. I. Pupin, Dr. Ludwik Silberstein. The afternoon program was as follows:

A Brief Survey of the Historical Development of Optical Science, J. P. C. Southall
Helmholtz's Early Work in Physics—The Conservation of Energy....Henry Crew
Helmholtz's Contributions to Physiological Optics.....L. T. Troland

Prof. Crew exhibited lantern slides showing Helmholtz at the time he wrote the essay on the Conservation of Energy (age 26) and also at later periods of his life.

At the evening session Prof. M. I. Pupin spoke informally and in a most interesting and delightful manner on his Personal Recollections of Helmholtz. Prof. E. L. Nichols, Prof. Ernest Merritt, Dr. Ludwik Silberstein, Mrs. Christine Ladd Franklin and Prof. C. R. Mann also spoke of their memories of Helmholtz as a teacher. Prof. Mann showed a lantern slide of a photograph which he himself made on July 7, 1894 showing Helmholtz at his lecture desk only a few days before his last illness.

The Helmholtz Memorial addresses will be published in the Journal of the Optical Society of America.

Various scientific societies were represented at the meeting by delegates as follows:

American Mathematical Society: Prof. A. S. Gale.

American Physical Society: Prof. M. I. Pupin, Dr. L. T. Troland, Prof. Henry Crew.

American Association for the Advancement of Science: Prof. M. I. Pupin.

New York Academy of Science: Prof. M. I. Pupin.

American Academy of Opthalmology & Oto-Laryngology: Dr. R. S. Lamb.

American Medical Association, Section of Opthalmology: Dr. W. B. Lancaster.

American Opthalmological Society: Dr. Lucien Howe, Dr. George S. Crampton.

Society of Illuminating Engineers: Dr. George S. Crampton.

American Psychological Association: Dr. L. T. Troland, Mr. Prentice Reeves, Prof. C. E. Ferree, Dr. P. W. Cobb.

The following papers were presented at the regular sessions of the Society on October 25 and 26.

Tuesday, Oct. 25, 9:00 A.M.

Vision and Physiologic Optics

Photo-Electric Potentials from the Retina, E. L. Chaffee and W. T. Bovie, Harvard

The photo-electric potentials from the retina are studied by the use of an improved measuring system utilizing two stages of vacuum tube amplification. This apparatus,

besides giving greater sensitivity, has the advantage of measuring the potentials produced in the retina irrespective of the resistance of the tissues.

A typical response of the retina, found by previous experimenters, when the retina is illuminated by a flash of light, consists of a small negative reaction followed by a rapidly rising positive potential which gradually subsides. There may follow a second very slow rise and fall of potential, lasting for several seconds for intense illumination. In the present work the response curves are found to be more complex, showing several maxima. Certain of these maxima are attributed to the scotopic mechanism, i.e., the rods, and others, to the photopic mechanism, i.e., the cones. Certain maxima of the observed curves are correlated with some psychological observations, such as recurrent vision, Bidwell's ghost, positive after image, etc.

Many curves have been taken showing the effect of varying the sensitivity and time of illumination. The laws connecting the height of the curves, considered to be a measure of the sensation, with intensity of illumination have been derived.

Intensity and Composition of Light and Size of Visual Angle in Relation to Important Ocular Functions. C. E. FERREE and GERTRUDE RAND, Bryn Mawr

The benefit of increase of illumination is shown for the following functions of importance to the working eye:—acuity, power to sustain acuity, speed of discrimination, and speed of adjustment of the eye for clear seeing at different distances. The benefit of the increase is considerably greater for eyes suffering from a slight defect of refraction than for the normal eye. A comparison is made of the effect of increase of intensity of illumination and increase in size of visual angle. The question of the most favorable intensity of illumination of test charts for different test purposes is discussed. The investigation is also extended to include the effect of variations in the composition of light on acuity, power to sustain acuity, and speed of discrimination.

A Theory of Intermittent Vision. HERBERT E. IVES, Western Electric Co., New York

An attempt to postulate a physical mechanism which will give the well-known logarithmic relations between critical frequency of disappearance of flicker and illumination and wave form of the stimulus. A system is studied consisting first of a photo-sensitive cell of considerable capacity, in which the material decomposed by light is replaced at a rate such that the equilibrium position is proportional to the logarithm of the intensity; second, of a conducting medium in which there occurs either a leakage or a regeneration of the conducted products of decomposition. If it is assumed that the perception of flicker is dependent on the attainment of a certain rate of change of the transmitted impression, this physical system will exhibit all the principal critical frequency relationships.

(Complete Paper, This Journal June, 1922)

An Analysis of the Visibility Curve in terms of the Weber Fechner Law and the Least Perceptible Brightness. ENOCH KARRER, Nela Research Laboratory

The possibility of getting an insight into some of the properties of the eye, by considering it as an instrument, has attracted the attention of many, among the first of whom are Fechner and Helmholtz.

Consider two instruments, designed to measure the same thing. If one instrument gives the same increase in deflection per unit of quantity measured at all points of the scale, as does the other (i.e., the sensitivity of the one is the same as that of the

other) and gives the same deflection under the reference condition as does the other (i.e., the same zero reading), then we may expect them to give the same deflection for the same arbitrary quantity measured.

The sensitivity is some function of the stimulus, while the deflection is the integral effect of the sensitivity at all points of the scale passed over. In case of the eye the sensitivity function may be obtained from the photometric sensibility (the least sensible increment of flux expressed in terms of the stimulating flux). We get then an expression for the sensation of brightness for any wave-length in terms of the flux and the photometric sensibility. For equal brightness at several wave-lengths there is a corresponding series of values of the flux. The visibility of radiation at any wave-length is inversely proportional to the flux when the brightness level is the same for all wave-lengths. We have then an expression for the visibility of radiation in terms of the photometric sensibility and several constants of the eye.

It is suggested that the determination of the least flux sensible to the eye, and of the photometric sensibility of eyes whose visibility curves are sought may be of great theoretical and perhaps practical value. It would for example be enlightening to find eyes having the same photometric sensibility and the same threshold visibility (the inverse of least sensible flux) and yet having under the same condition quite different visibility curves.

A Quantitative Determination of the Inherent Saturation of Spectral Colors. L. T. TROLAND, Harvard

Introspection, as well as the facts of color mixture, indicates that the spectral colors, although of equal physical purity, are of quite different subjective saturation. The same fact is strongly indicated by the differences between their flicker photometer frequencies at equal brightness. The flicker photometer frequency value presumably depends upon the degree of difference existing between the given color and the white with which it is rapidly alternated. The method of the present work was to mix with the various spectral lights the amount of white light required to give them all the same flicker photometer frequency. The differences between the amount of white required for this purpose by different spectral colors may be taken as indices of their respective saturations. The white employed as a standard and for admixtures was a color match with noon sunlight. Parallel measurements of the flicker photometer frequencies of the various spectral colors without admixture of white were made. The results show a minimum of saturation for the yellow, rapidly increasing toward either end of the spectrum, the blue showing the highest value and requiring an admixture of about eighty per cent white to bring it to the same frequency value as the yellow. The bearing of the results upon color sensation theories is discussed.

The Interrelations of Brilliance and Chroma Studied by a Flicker Technique. L. T. TROLAND AND C. H. LANGFORD, Harvard

Advocates, such as Abney, of the Young-Helmholtz Theory of Color Vision have assumed that brilliance, as represented by the visibility curve, is the sum of three separate chromatic excitations, as represented in the color sensation curves. This view is opposed by the Hering and other theories which suppose that at least part of the brilliance value is due to a mechanism which is independent of the chromatic processes. The present work endeavors to meet certain criticisms which render the

experiments of Abney and others bearing upon this problem of doubtful significance. The general method which was employed was to fatigue the retina with a spectral red stimulus, to measure the degree of this fatigue in terms of brightness, and then to repeat these brightness measurements with the same fatigue conditions projected upon a minus red reacting stimulus field. The Young-Helmholtz interpretation demands distinctly different results for the two reacting stimuli, while the Hering view requires less difference, or possibly identical results, for the two. In order to reduce the uncertainties produced by the extreme saturation contrast generated in the experiment the flicker method was employed for establishing the brilliance equations. The results show that brilliance has, apparently, much less dependence upon the chromatic excitation than is ordinarily assumed in interpretations of the Young-Helmholtz theory. The independence, however, is not complete. The definite bearing of the results upon the three sensation curves of Abney and of König, as well as upon hypotheses similar to that of Hering is discussed.

COLORIMETRIC MEASUREMENTS

A Proposed Standard Method of Colorimetry. HERBERT E. IVES, Western Electric Co., New York

A method of color measurement is suggested which consists essentially of the spectro-photometry of adjacent patches of the spectrum, each patch of a width inversely as the hue sensibility, and the number of patches dictated by the kind of color and degree of accuracy required. An instrument is described by which the measurements can be made, and in which the color can be reproduced for study.

(Complete Paper, This Journal, Nov. 1921)

Accuracy in Color Matching. W. E. FORSYTHE, Nela Research Laboratory

Two tests have been made of the accuracy of color matching with an ordinary Lummer-Brodhun contrast photometer.

In the first test, several observers made color matches using two tungsten lamps so set that there was an illumination of 5.1 foot-candles on the photometer screen. It was found that experienced observers agreed in their setting to an accuracy of about 3°K, while inexperienced observers differed by only about 5°K.

There was also a test made to see if there was any difference in the accuracy depending upon the illumination on the photometer screen. Tests were made by several observers for illumination varying from 1.8 to 45 foot candles. There was no very definite difference in accuracy found for the different illuminations.

A lamp that had been calibrated for a color temperature of about 3000°K was sent to the Bureau of Standards and compared with their color standard. It was found that at this color temperature the two scales were in agreement within the accuracy claimed.

Measurement of the Color Temperature of the More Efficient Artificial Light Sources by the Method of Rotatory Dispersion. IRWIN G. PRIEST, National Bureau of Standards

A description of measurements of color temperatures from 3000° to 4000°K by means of the rotatory dispersion apparatus previously described by the author (J. Op. Soc. Am., 5, p. 179, Fig. 2; March, 1921). Particular attention will be given to (1) a consideration of the fundamental standard of color temperature for such measure-

ments, (2) a more careful temperature calibration of the apparatus than that previously published, (J. Op. Soc. Am., 5, p. 182, Fig. 6). Experimental data will be given: (1) on the color temperature of the photometric standard gas-filled tungsten lamp as a function of efficiency from normal efficiency up to the failure of the filament; (2) on the color temperature of the crater of the carbon arc; and (3) on comparisons with the color temperature scale of the Nela Research Laboratory.

(Complete Paper, This Journal, Jan. 1922)

WEDNESDAY, OCT. 26, 9:00 A. M.

MISCELLANEOUS PAPERS

The Blue Glow. E. L. NICHOLS and H. L. HOWES, Cornell

The effect described in this paper is observable when certain oxides, notably MgO, CaO, BeO, ZrO, SiO_2 and Al_2O_3 are heated to incandescence. These transparent solids are very poor radiators. Below 800° C for example their radiation, as viewed through a red screen is a small fraction of one per cent of that emitted by a black body at the same temperature. Viewed through a blue screen, however, these oxides are found to radiate many times as intensely as a black body, so that they are visible at temperatures so low that a black body, thus viewed, is still invisible. At a "red heat," in other words, they appear blue.

That the blue glow is a case of luminescence is evident from the following considerations:

(1) For certain regions of the spectrum, as just stated, the radiation is in excess,—often greatly in excess of what a black body emits for the same wave-length and temperature.

(2) The change of intensity with temperature does not follow the laws of temperature-radiation. The effect occurs only within a definite and rather narrow temperature range and is confined to a definite band in the blue-violet region of the spectrum.

(3) a. —The effect is subject to fatigue.

b.—It is profoundly modified by previous heat treatment of the oxide.

c.—It is profoundly modified by the presence or absence of certain activating materials in which respect it resembles kathode-luminescence.

(4) The effect is much more marked when heating occurs by means of the H-O flame than otherwise and is affected as to its intensity by the excess or lack of free oxygen. In this respect it resembles the numerous types of luminescence which are known to involve reduction and oxidation.

The oxides under consideration undergo marked temporary changes in their physical properties when carried through the range of temperatures (500° to 1200° or 1400°) within which they show the blue glow. For example they become opaque, their power of temperature-radiation increases from a negligible value to almost unity and their electrical resistance—as is well known from the measurements of Somerville and others—falls from near infinity to the small values characteristic of metallic conductors.

The blue glow is a special case of the general phenomenon of luminescence of incandescent bodies now under investigation by the present authors.

(Complete Paper. This Journal, Jan. 1922)

The Optical Properties of a Cylindrical Enclosure with Specularly Reflecting Walls. HERBERT E. IVES, Western Electric Company, New York.

A study of the multiple reflections occurring inside a hollow cylinder, such as a tube of platinum foil, with a narrow viewing slit in the side. For certain angles of observation such an enclosure is a practical black body, because the reflected images escaping through the slit are so diminished by successive reflection as to be of negligible intensity. A graphical solution is constructed by which the number, position, and size of the reflected images for any width of viewing slit may be found. From this can be found the best angle of observation and size of slit to choose in using an incandescent cylinder as a black body for such a purpose as determining the melting point of the material used.

The Relation Between Gloss and the Reflection Characteristics of Surfaces. LOYD A. JONES and M. F. FILLIUS, Eastman Research Laboratory.

A method and instrument for the measurement of the gloss of a reflecting surface is described. The measurement of gloss is based upon a comparison of the intensity of the diffuse and specularly reflected light. The surface considered is illuminated by a parallel beam of light incident at 45° from the normal. Under these conditions the ratio of the brightnesses of the surface when viewed at an angle of 45° on the other side of the normal and normally is taken as a measure of gloss. Numerous data on various well-known surfaces and upon a number of different photographic papers and stocks are given. The relation between the gloss values thus obtained and the complete distribution curve of light reflected from these surfaces is considered.

(Complete Paper, This Journal, March, 1922)

The Graininess of Photographic Materials. LOYD A. JONES and ARTHUR C. HARDY, Eastman Research Laboratory.

The theoretical considerations upon which the measurement and numerical specification of graininess are based are discussed. The previous work on the subject is reviewed briefly. Differences in the details of procedure used formerly and those adopted after further investigation of the problem are discussed. From a consideration of the data obtained it is shown that the precision obtainable under favorable conditions is of the order of plus or minus 2 per cent. Numerous data relating to the influence upon graininess of such factors as the nature of the photographic material, the constitution, concentration, temperature of the developing solution, and the time of development are given. The graininess resulting when a negative of known graininess is printed upon a positive material is also considered.

(Complete Paper, J. Frank Inst.)

The Design of Aspherical Lens Surfaces. P. G. NUTTING.

The use of a new system of ray triangulation leads to the differential equation of a lens surface which will produce a specified convergence in the refracted rays. The method will be outlined and several solutions and practical applications will be discussed.

On the Distribution of Light in Planes Above and Below the Image Plane in the Microscope. FRED E. WRIGHT, Carnegie Geophysical Laboratory.

In microscope image formation diffraction plays an important part. The phenomena produced by the diffraction have been investigated repeatedly, both experi-

mentally and mathematically; but in the mathematical treatment the effects resulting from diffraction in the image plane, conjugate to the object plane, alone have been considered. But in practical miscroscopic work, especially with the petrological microscope, the phenomena which develop at the margins of fine grains on raising or lowering the tube are of importance, particularly in the determination of relative refringence. In the present paper these phenomena are considered in some detail, and the bearing of the several factors, diffraction, interference, refraction, and reflection is emphasized.

The Factors Underlying the Measurement of Refractive Indices by the Immersion Method. FRED E. WRIGHT, Carnegie Geophysical Laboratory.

In this paper a brief discussion is given of the rôles played by refraction, dispersion, reflection, diffraction, and interference in the phenomena observed in the measurement of refractive indices by the immersion method; also a statement of the degrees of accuracy obtainable by this method, especially in its application to petrographic microscopic work.

Some Thermal Effects Observed in Chilled Glass. A. Q. TOOL and C. G. EICHLIN, National Bureau of Standards.

Glass, cooled rapidly through the temperature range of the endothermic effect observed on heating it, shows, on reheating, an exothermic effect also, which begins at a much lower temperature than the normal endothermic effect. It is a difficult matter to determine whether this exothermic effect is due to the dissipation of the strains caused by the chilling, or to some molecular regrouping delayed by it. On certain assumptions the maximum effect to be expected, resulting from a stress relaxation, would differ not widely from the exothermic effects observed. Certain phases of these effects, however, indicate that this can scarcely be their whole cause. This paper deals with results obtained in an investigation to determine the cause of these effects and the manner in which they are modified by different methods of re-annealing the glass until it becomes normal again.

A New X-Ray Diffraction Apparatus. WHEELER P. DAVEY, Research Laboratory, General Electric Co.

An apparatus is described by means of which the X-Ray diffraction pattern of fifteen powdered crystals can be taken simultaneously. A scale is also described by which the distance, in Angstrom units, between planes in a crystal may be read off directly from the diffraction pattern. The uses of such an apparatus in research and in factory-control work are pointed out, and a method is briefly described by which the structure of certain classes of crystals may be rapidly determined by the aid of plots, on a logarithmic scale, of the diffraction patterns.

(Complete Paper. This Journal, Nov. 1921)

APPARATUS DEMONSTRATIONS

Rotating Photometric Sectors of Adjustable Transmission While in Motion. CARL W. KEUFFEL and C. D. HILLMAN, Keuffel and Esser Co.

A description of apparatus recently designed and constructed by Keuffel and Esser, illustrated by demonstration of the apparatus. The combination of one of these sectors with a spectrometer forms an accurate direct-reading spectrophotometer of very simple design.

Euscope. WILLIAM G. EXTON, Prudential Laboratory.

The Euscope affords microscopy such physiological advantages as natural posture of body and head, normal function of ocular musculature, retinal relief, single binocular vision and enlarged images. It is a modified pyramidal viewing box mounted on a stand. The smaller or ocular end is shielded from side light and fitted with diaphragm and lens-holding device in front of which, on its floor, is an aperture for the removable mount of a reflecting prism. The design of the screen end is adapted for projection and photography. Drawing, demonstrating and photography are unusually convenient. Orthoscopic images or increased magnifications may be had with proper lenses, and good results are obtainable with other optical instruments as well as the microscope without change of technique. (Apparatus exhibited.)

Turbidimeter. WILLIAM G. EXTON, Prudential Laboratory.

Means of measuring the degree of cloudiness of a suspension are of interest because the fields of nephelometry and trubidimetry are constantly expanding. Such estimations are now made by comparing the unknown with freshly prepared standards which are read against themselves before matching, etc. In trying to find a simpler and more direct method for quantitating albumin in urine, experiments were made with a view to utilizing the obscuring power of a column of cloudy liquid as a measure of its turbidity. With suitable illumination and targets this was found practicable, and exeriments indicated that the light extinguishing power of the column of cloudy liquid afforded perhaps a less subjective method. Laboratory and clinical models of self-standard turbidimeters have been constructed which work on either principle by giving vernier readings of the depth in millimeters at which a column of cloudy liquid obscures a design or extinguishes a light. Different interchangeable targets permit specialized application, allowance for color difference and checking results. (Apparatus exhibited.)

On Tuesday evening, October 25, visiting members were guests at a dinner entertainment given by the Rochester Section of the Society.

The very well conducted trips through the Research Laboratories of the Eastman Kodak Company and the glass plant, optical shops and observatory of Bausch and Lomb were also much appreciated by the visiting members.

The Rochester Section was given a hearty vote of thanks for its hospitality and the many courtesies extended during this very successful meeting.

Forty new members were elected. The membership is now about three hundred.

The Society's intention to cultivate actively the field of physiological optics was indicated by the following resolution, adopted October 26, 1921:—

WHEREAS, the Optical Society of America is devoted to the science of optics, pure and applied, a subdivision of which is the subject of Physiological Optics, with the several contributory sciences of physiology, psychology, physics and chemistry, representatives of which sciences have at the present time no common meeting ground for the discussion of problems of vision of mutual interest and

WHEREAS, the National Research Council, through its Committee on Physiological Optics, has recommended to the Optical Society of America the taking of such steps as may be necessary to further and encourage cooperative efforts in research in vision and allied phenomena, therefore be it

RESOLVED, that the Optical Society of America does hereby signify its intention of devoting one or more sessions of each annual meeting to papers on Physiological Optics and other appropriate subjects related to vision, and

Resolved that there be and hereby is established by the Society a Standing Committee of three the duty of which shall be

(1) to prepare the program of the sessions on Vision,

(2) to coordinate the work of the Society in this field with the work of other Societies and

(3) to recommend, from time to time, such further steps as may be deemed effective in encouraging research in Physiological Optics and allied problems. And,

RESOLVED, further that the Optical Society, through its Committee on Physiological Optics, shall invite all those interested in research on Vision and allied fields to participate actively in these sessions.

The next meeting will be held at the National Bureau of Standards in Washington in the latter part of October 1922. It is tentatively planned to hold an exhibition of optical instruments in connection with this meeting.

IRWIN G. PRIEST, *Secretary*.

OPTICAL SOCIETY OF AMERICA

NEW MEMBERS

The following new members have been duly elected since October 26, 1921.

REGULAR

Herbert P. Hollnagel, 22 Hampden St., Swampscott, Mass.

Herbert M. Reese, Physics Bldg., Columbia, Mo.

Conway D. Hillman, 89 Harrison St., East Orange, N. J.

O. E. Conklin, Parlin, N. J.

Edward H. Kurth, Graduate College, Princeton University, Princeton, N. J.

Anthony Zeleny, University of Minnesota, Minneapolis, Minn.

Hans Harting, Albrechtstr. 12, Berlin-Schlachtensee, Germany.

L. F. Yntema, 357 Chemistry, University of Illinois, Urbana, Ill.

Royal A. Porter, 861 Ostrom Ave., Syracuse, N. Y.

Henri Pieron, à la Sorbonne, Paris, France.

E. LeRoy Ryer, 9 East 46 St., New York City.

Alpheus W. Smith, Dept. of Physics, Ohio State Univ., Columbus, Ohio.

Frederic C. Blake, 2107 Iuka Ave., Columbus, Ohio.

R. W. Cheshire, Admiralty Research Laboratory, Teddington, Middlesex, England.

James E. Ives, Office of Industrial Hygiene & Sanitation, U. S. Public Health Service, Washington, D. C.

ASSOCIATE

Olive M. Lammert, Vassar College, Poughkeepsie, N. Y.

W. H. Fulweiler, 319 Arch St., Philadelphia, Pa.

Madelaine Ray Brown, 13 Charles Field St., Providence, R. I.

ASSOCIATE (continued)

George W. Sherman, Jr., No. 4 Murdock Apts., West LaFayette, Indiana.
Thomas R. Harrison, c/o Champion Porcelain Co., Detroit, Mich.
Donald M. Smith, Dept. of Physics, Iowa State College, Ames, Ia.
Casper L. Cottrell, Rockefeller Hall, Ithaca, N. Y.
John Arthur Spengler, 73 Seneca St., Geneva, N. Y.

IRWIN G. PRIEST,
Secretary.

WASHINGTON,
MAY 22, 1922